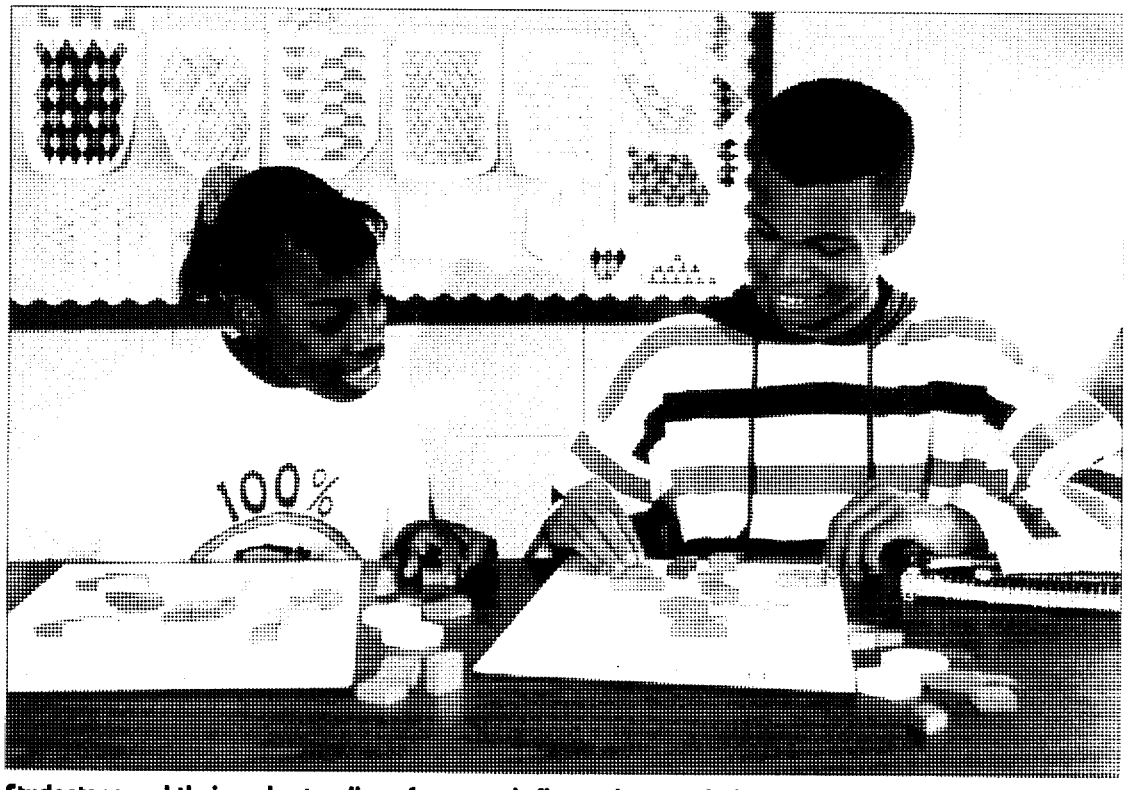
Jean M. Shaw,
Conn Thomas,
Ann Hoffman, and
Janis Bulgren

USING CONCEPT DIAGRAMS TO PROMOTE UNDERSTANDING IN GEOMETRY



Students reveal their understanding of geometric figures by completing a concept diagram.

The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) and the van Hiele model for geometric thought (Crowley 1987) advocate increasing students' understanding of geometric properties and relationships as they enter the intermediate and middle grades. Such understanding requires that students comprehend the attributes of geometric figures and develop clear concepts of those figures. Concepts are the building blocks, or foundations, on which more complex ideas are established. Students who understand geometric concepts are better equipped to generalize and to transfer their knowledge than students who have merely memorized definitions.

Students develop concepts in many ways. Focused and informal discussions and group work assist learners in dealing with concepts. Experiences with manipulatives and models help them develop and assimilate geometric ideas. Attention to examples and nonexamples enhances students' abilities to refine their thinking about concepts.

Jean Shaw teaches at the University of Mississippi, University, MS 38677. She is interested in developing understanding and confidence in mathematics students of all ages. Conn Thomas is the director of the Panhandle Alternative Certification for Educators (PACE) at West Texas State University, Canyon, TX 79016-0208. He has a special interest in helping slow learners in all areas. Ann Hoffman is an educational consultant in Cedar Rapids, IA 52405. She is interested in learning strategies for middle- and upper-grade students. Janis Bulgren is a research associate at the Institute for Research in Learning Disabilities, University of Kansas, Lawrence, KS 66045. Her special interests are learning strategies and promoting academic success.

Using Concept Diagrams

Using concept diagrams is one way to promote understanding the ideas of geometry. Such diagrams help students construct and clarify concepts through definitions, characteristics, and examples. These diagrams offer a framework that enables students to organize their thinking. The parts of a concept diagram are described as follows (Bulgren 1986).

- Concept name: The concept name is a word or group of words that identify a category or class.
- Definition: The definition states characteristics that are always present in the concept and shows how they relate and connect.
- Characteristics: Three categories of characteristics are included in the concept diagram. First are those characteristics that are *always present*—essentially, the definition. Second are those characteristics that are *sometimes, but not always, present*. Third are those characteristics that are *never present*. The characteristics used in a concept diagram must be meaningful and varied. Examining the characteristics assists students in clarifying the concepts.
- Examples and nonexamples: Examples further clarify and enhance an understanding of the concepts. Each example must illustrate the characteristics identified in the "always" category. Examples are used to show learners what a concept *is;* nonexamples help to show them what a concept *is not*. **Figures 1** and **2** are examples of concept diagrams.

If middle- and upper-grades students are already

Students compare and contrast their examples

FIGURE 1

Deductive-reasoning concept diagram for *square*

Concept name: Square

Definition: A square is a four-sided polygon with equal sides and right angles.

Characteristics present in the concept:

Always	Sometimes	Never
4 equal sides	small	round
4 right angles	large	unequal sides
polygon	horizontal sides	less than 4 sides
	vertical sides	more than 4 sides
	medium size	open

Example: Nonexample:

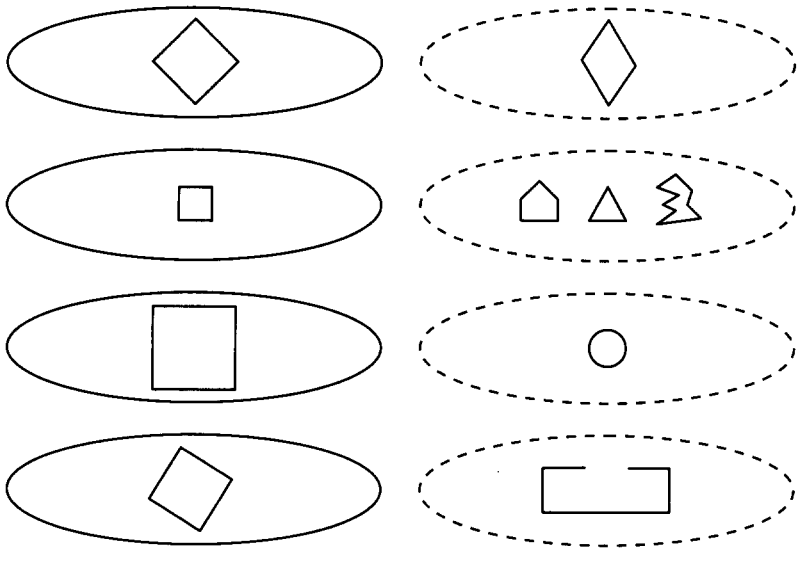


FIGURE 2

Deductive-reasoning concept diagram for *prism*

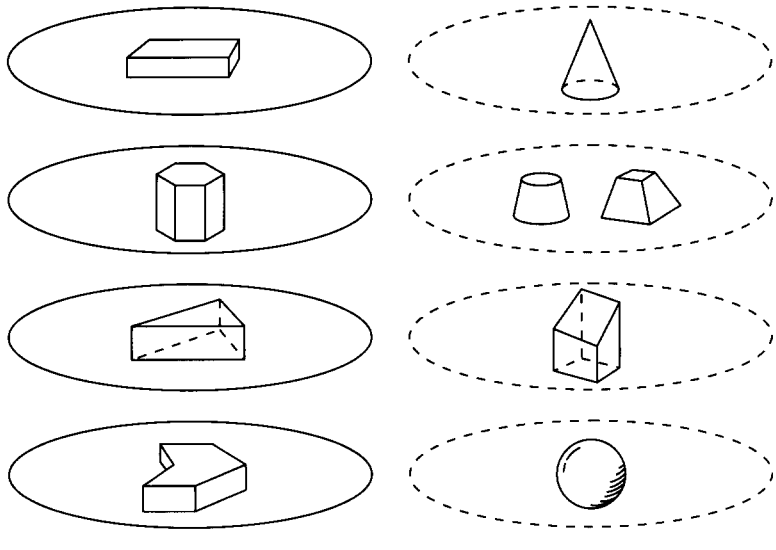
Concept name: Prism

Definition: A prism has 2 bases that are congruent polygons. The bases are parallel planes.

Characteristics present in the concept:

Always	Sometimes	Never
2 bases	square bases	bases are circles
bases are $\cong$	hexagonal bases	only 1 base
bases are parallel	"short"	has bases that meet
	"tall"	has bases of different
	regular polygon bases	sizes
	not regular polygon bases	

Example: Nonexample:



somewhat familiar with the definitions for geometric terms, teachers can encourage *deductive reasoning,* as illustrated in **figures 1** and **2.** Concept names and definitions are presented to students, or students use the definitions found in their textbooks. For example, the definitions given in **figures 2** and **4** come from *The Mathematical Experience, Level 7* (1992, 448). After reviewing a definition, students work in groups and make a concept diagram, filling in the characteristics and the examples and nonexamples. The students then check the characteristics and examples by reviewing the definition. They will note that the "always" characteristics are found in the definition. Their examples may also include some characteristics from the "sometimes" category but no characteristics from the "never" category.

To further check their examples against the characteristics, students draw a connecting line from each example to the characteristics, as shown for one example in **figures 3** and **4.** When checking their examples, students should mark plus signs on the "always" characteristics and minus signs on the "never" characteristics. They should examine the "sometimes" characteristics and mark plus signs or minus signs to indicate which characteristics are present. As students compare and contrast their examples, they will recognize the variations and the uniqueness of each example. They will see that not all squares are the same size and that not all

FIGURE 3

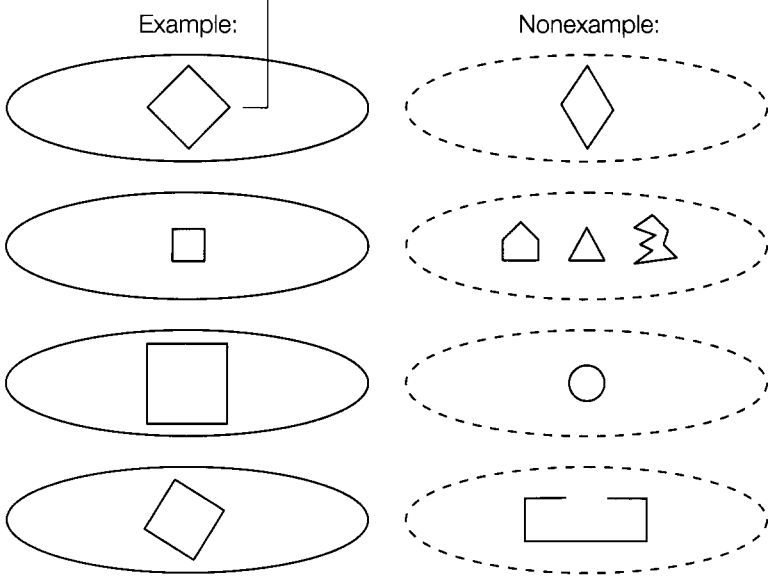
Deductive-reasoning concept diagram for *square*

Concept name: Square

Definition: A square is a four-sided polygon with equal sides and right angles.

Characteristics present in the concept:

Always	Sometimes	Never
+ 4 equal sides	- small	- round
+ 4 right angles	- large	- unequal sides
+ polygon	- horizontal sides	- less than 4 sides
	- vertical sides	- more than 4 sides
	+ medium size	- open



squares have the same orientation in the plane. All the examples of squares, however, exhibit the "always" characteristics and none of the "never" characteristics.

At this point, the diagram serves as a basis for communication. Groups of students compare their examples and nonexamples and their use of plus and minus signs. Groups compare their pencil-and-paper diagrams. They gather around and talk through their diagram using manipulatives or models. If each group draws a diagram on a transparency, group members can present and compare their results as the whole class looks at each diagram. Presenters could invite their classmates to evaluate their ideas by testing them against models and sketches. Thus the concept diagram constitutes a basis for self-verification, testing ideas, and checking for understanding.

Teachers can also have students use *inductive reasoning* to generate their own definitions. An inductive format is helpful when students are somewhat familiar with examples of a concept but are less knowledgeable about the definition. The inductive format, shown in **figure 5,** is created by reversing the order of the deductive-reasoning concept diagram. Examples and nonexamples are given first, then the three types of characteristics are generated. Students next build and word the definition, deriving it from the characteristics

Diagrams help organize key concepts

Relationships among definitions, characteristics, and examples are evident

FIGURE 4

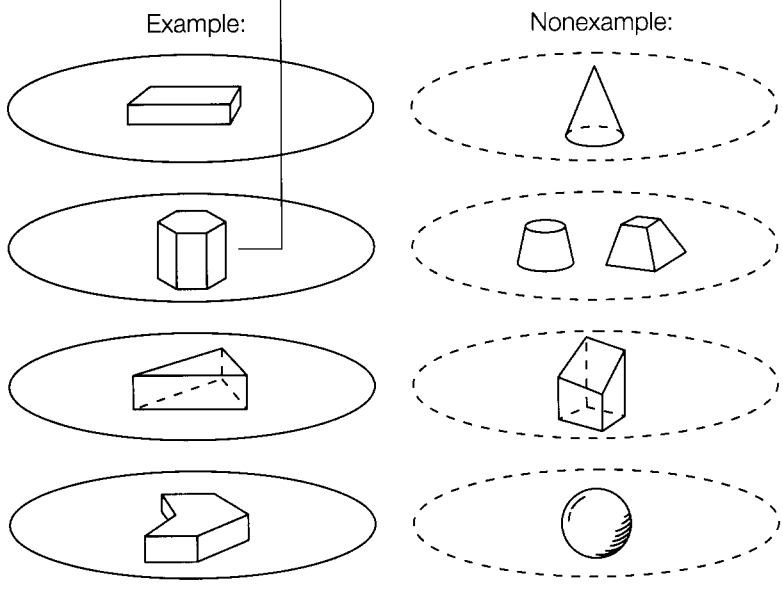
Deductive-reasoning concept diagram for *prism*

Concept Name: Prism

Definition: A prism has 2 bases that are congruent polygons. The bases are parallel planes.

Characteristics present in the concept:

Always	Sometimes	Never
+ 2 bases	- square bases	- bases are circles
+ bases are ≅	+ hexagonal bases	- only 1 base
+ bases are parallel	- "short"	- has bases that meet
	+ "tall"	- has bases of different sizes
	+ regular polygon bases	
	- not regular polygon bases	



that are always present. This process allows students to examine the examples and characteristics closely and create a definition and a concept name.

Advantages of Using Concept Diagrams in Mathematics

The use of concept diagrams can benefit both teachers and students. For teachers, thinking about a diagram clarifies the concept before they present it to students. The diagram assists the teacher in organizing examples and forming questions to prompt students' thought. The process also helps the teacher to think about essential information and to organize learning examples that help students develop and internalize the concept. In addition, using concept diagrams allows the teacher to observe students' ways of reasoning and their depth of understanding. Misconceptions may show up as students generate, discuss, and verify concept diagrams. Teachers can then clear up those misconceptions.

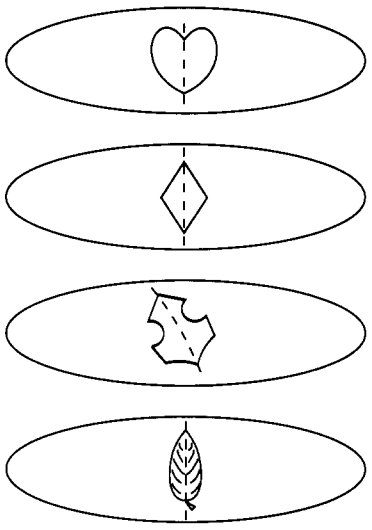
Concept diagrams involve intermediate-level students in reasoning processes that are vital to mathematical understanding. Diagrams help students visually organize important concepts and share their thinking with others. The diagram's concise format appeals to students faced with acquiring many new concepts. Relationships among definitions, characteristics, and examples are clearly evident. The diagram complements and supplements work with manipulatives by helping students recall, organize,

FIGURE 5

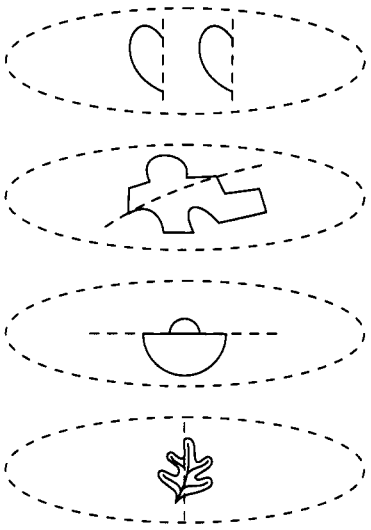
Inductive-reasoning concept diagram for *line symmetry*

Concept name: Line symmetry

Example:



Nonexample:



Characteristics present in the concept:

Always	Sometimes	Never
can be folded on a line to make halves	has curved parts	without a line of symmetry
halves are mirrored images	has straight parts	has unmirrored halves
	occurs in nature	has parts other than halves
	occurs in people's designs	
	has horizontal lines	
	has vertical lines	

Definition: A figure has line symmetry if it can be folded on a line to make halves that are mirrored images.

and comprehend the knowledge gained in working with physical objects.

Part of a student's becoming "mathematically independent" is reflecting on and clarifying her or his own reasoning. As students check all the characteristics and examples in their concept diagrams, they show and discuss the uniqueness and appropriateness of their examples, thus taking part in clear and logical thinking processes.

Conclusion

The inductive and deductive formats in concept diagrams help students picture ideas in organized ways. Concept diagrams can be used with upper elementary and secondary students in any aspect of geometry. Teachers will find ways to employ concept diagrams to enhance students' thinking and reasoning abilities in other areas of mathematics as well as in other subjects that require the understanding of concepts.

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Students take part in clear and logical thinking processes