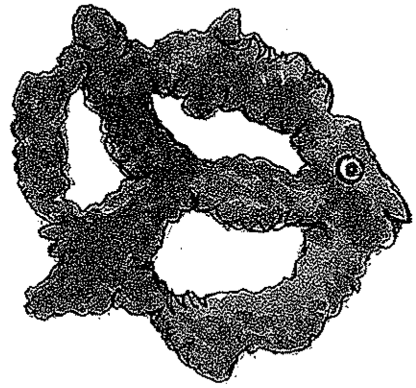
One Fish, Two Fish, Pretzel Fish



Learning Estimation and Other Advanced Mathematics Concepts in an Inclusive Class

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Cooperative learning

Estimation techniques

Calculators

Graphic organizers

Links to prior knowledge

Real-life problems

Cue-do-review

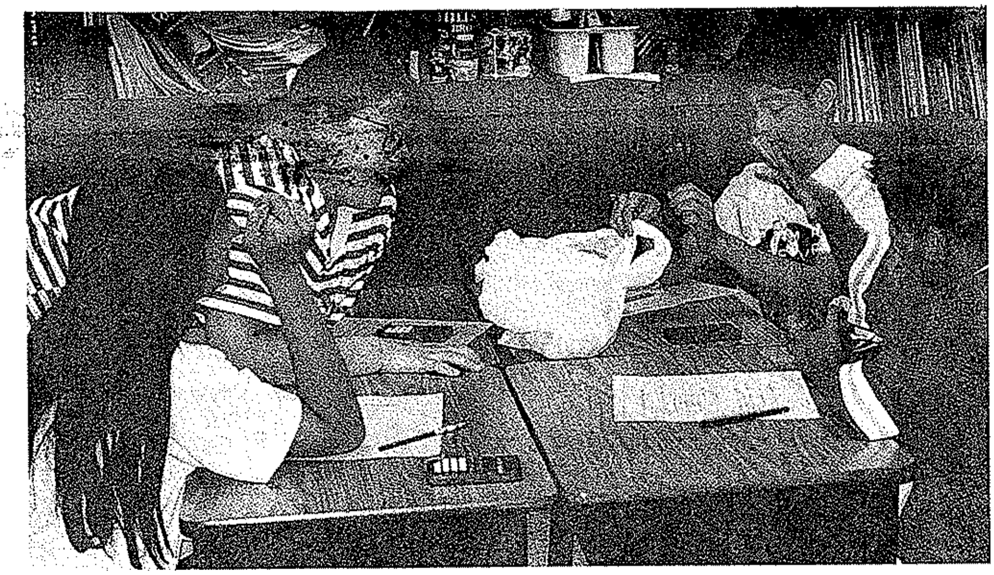
Using a combination of research-based approaches like these, a team of teachers successfully taught fifth-grade students in an inclusive classroom to use various strategies to learn advanced mathematics concepts and skills. With the help of a mathematics consultant, two teachers (a general and a special education teacher) used the capture-recapture statistical estimation method (Landwehr, Swift, & Watkins, 1987), as well as the Lesson Organizer Routine (Lenz, Marrs, Schumaker, & Deshler, 1993).

This article shows how teachers can work together to help students in inclusive classrooms learn advanced mathematical content and skills. The keys to the success of the students are teamwork, research-based strategies, and student engagement and ownership.

Providing effective mathematics instruction for diverse and inclusive groups of students required systematic planning and powerful instruction.

Challenges—and Teamwork—in an Urban School

The fifth-grade classroom teacher and the mathematics consultant had been participating in a Minority Mathematics and Science Education Cooperative, an Eisenhower grant, over a 2-year period. The goal of the project was to improve student mathematics and science achievement in minority school districts. As a result of being involved with the grant—and knowing that many students required special education and English-as-a-second-language (ESL) services—the teacher questioned her ability to provide effective mathematics lessons that would meet the varying instructional needs of all the students assigned to her classroom without watering down the district's mathematics curriculum.



Students' opportunities to learn mathematical and statistical concepts can be improved with the combined use of projects, calculators, and cooperative learning groups.

The school was in a largely Hispanic (>90%) urban school district in the greater area of San Antonio, Texas. Here are the demographics of the fifth-grade class:

- Twenty-one students, including 8 males and 13 females, ranged in age from 10 to 12 years old.
- The group included 19 students with Hispanic surnames, 1 Caucasian student, and 1 Middle Eastern student.
- Eleven students were judged to be culturally and linguistically diverse and demonstrated limited English proficiency.
- Two students were receiving inclusive special education services under the category of serious emotional disturbance.
- The class included 6 students with specific learning disabilities and 4 students with speech and communication disorders.
- Six of the students with disabilities were concurrently receiving ESL instruction.

The school had recently reorganized its special education services to include a *continuum of services model* and inclusion placements, based on the individualized education program (IEP) of each student with disabilities. In addition to working with classroom teachers in inclusionary classrooms, the special education teacher was still expected to provide both remedial and corrective instruction in a resource room. Thus, this special educator had only minimal contact with the fifth-grade teacher and her special education students.

All the teachers wanted to avoid ineffective practices. For example, in an attempt to meet the varying and diverse needs of their students, many teachers use one of these commonly tried solutions.

- They try to gear mathematics lessons and activities to the mean or average of the class.
- They form multiple instructional groups within a class period in the effort to provide differing levels of instruction, based on student needs or mathematical skill levels.

Unfortunately, although these approaches are intuitively practical and appear to make sense, research has shown that they are not very effective, particularly for students with disabilities (Bottge & Hasselbring, 1993; Brosnan, 1997; Hutchinson, 1987; Maccini & Hughes, 1997; Miller, Mercer, & Dillon, 1992).

To gain additional support and suggestions for both general and special educators, the mathematics consultant contacted a special education consultant not affiliated with the school district or the grant. Together, these professionals committed themselves to finding ways to help the fifth-grade teacher meet the mathematics needs of her students.

Teachers in inclusive classrooms need to provide effective mathematics lessons that will meet the varying instructional needs of all students, without watering down the district's mathematics curriculum.

Seeking Solutions to Meet the Challenges

To address the instructional challenges presented by the students assigned to the fifth-grade inclusive classroom, the teacher, the mathematics consultant, and the special education consultant followed a plan of action:

1. They first reviewed the learning characteristics, mathematics achievement, and skill levels of the group. They examined cumulative records, classroom products and test scores, and the teacher's observations about individual student performance.
2. They examined the current unit of the school district's mathematics curriculum: "Understanding and Using Statistics." They wanted to determine the content to be covered in the unit and to identify the learning processes the students would need to use to succeed in learning the content of the unit.
3. The mathematics consultant suggested that as a *supplement to the curriculum*, they could use the "capture-recapture method" described in *Exploring Surveys and Information from Samples* (Landwehr et al., 1987). In real life, statisticians use capture-recapture methods to estimate populations that are practically impossible to count, such as wildlife. After examining the profiles of the students, the teachers determined

Figure 1. Student Worksheet

Estimating Populations

NAME:______________________

After you have taken your sample of Goldfish, complete the following table:

N = total number of fish in your sample

20 = number of marked fish in the paper sack

________ = number of marked (pretzel) fish in your sample

________ = total number of fish in your sample

Solve for N in the following proportion:

$$\frac{20}{N} = \frac{\text{number of marked (pretzel) fish in your sample}}{\text{total number of fish in your sample}}$$

Your **estimate** for the number of fish in the paper sack is ____________.

The classes **estimate** for the number of fish in a sack is ____________.

The actual number of fish in a package is _________. (Your teacher will tell you this number.)

What have you learned from this activity?

When could this type simulation be used in real life other than estimating fish?

they needed to introduce a majority of the students in the class to writing ratios and solving proportions—to enable them to apply the capture-recapture method. To address this prerequisite, the mathematics consultant and the teachers developed an activity and accompanying student worksheet to introduce the concept of ratios and the process for solving proportions (see Figure 1).

4. The teachers and the mathematics consultant agreed that *calculators* would be used by those students having problems making numerical calculations (see box, "Time to Calculate!").
5. The three professionals decided the learning outcomes for the unit would be more meaningful for the students if they could use *real-world situations or real data sets* to teach the concepts embedded in the unit. Real data sets are authentic, interesting, and relevant; provide students with background information; and allow students to perform multiple analyses.

   In many mathematics programs, teachers often use "canned data"; as a result, many students do not develop a real sense of the uses of statistics (Scheaffer, 1990; Singer & Willett, 1990). Recent studies have also shown that activity-based learning in the area of statistics is desired for deeper understanding of the concepts (Gnanadesikan, Schaefer, Watkins, & Witmer, 1997; Mittag & Eltinge, 1998; Taylor, 1993).
6. The mathematics consultant argued that in addition to the use of real data sets, students' opportunities to learn the mathematical and statistical concepts of the unit would be improved with the combined use of *projects, automated computation (calculators),* and *cooperative learning groups.*

The teachers agreed that calculators would be needed by those students having problems making numerical calculations.

In real life, statisticians use capture-recapture methods to estimate populations that are practically impossible to count, such as wildlife. The teachers decided to incorporate real-life situations into their lessons.

Thus, the professionals agreed that all these approaches and methods would be incorporated into the lessons covering the unit.

7. Given the complexity of the unit and the students' learning needs, the special education consultant suggested they should also explore a research-based approach to teaching called *content enhancement* developed at the University of Kansas Center for Research on Learning. This approach was specifically designed to be an

Time to Calculate!

Here are some research-based reasons for using calculators in math classrooms:

- Calculators provide *all* the students practice and success in calculating ratios and solving proportion problems.
- Calculators are effective for teaching advanced concepts, and they de-emphasize number crunching for students (Gilchrist, 1986).
- Calculators make complex calculations less tedious for students (Harvey, Waits, & Demana, 1995; Quesada, 1994).
- Calculators allow for the use of real data sets (Durham & Dick, 1994).
- Calculator use increases student confidence, enthusiasm, and number sense (Campbell & Stewart, 1993).

effective and efficient way of teaching academically diverse groups of students, as follows:

- The content enhancement approach values and addresses both group and individual needs of students.
- The approach maintains the integrity of the content.
- Content enhancement selects and transforms the important features of the content or skills to be taught in a manner that promotes student learning.
- Using this approach, teachers provide instruction in a partnership with students.

8. After reviewing literature on content enhancement, the three professionals found that this approach would provide them with a system for making decisions about what content or skills to teach. Specifically, they selected *The Lesson Organizer Routine* (Lenz et al., 1993). They determined that the routine would help them organize and convert the content of the unit into understandable formats, and it would provide the teacher with specific steps to follow in presenting the content in ways that would help *all* of the students learn. Likewise, they believed the routine would help them with the following instructional tasks:
   - Think intensively about what was critical for the students to know or learn.
   - Choose the major concepts that would make the related details, facts, or procedures taught in the unit more cohesive.
   - Pinpoint the specific relationships among the concepts embedded in the unit that they wanted the students to recognize.
   - Choose and construct instructional devices, materials, and activities that would *enhance* the content and concepts presented in the unit (i.e., making the content, concepts, or skills understandable and easy to remember).
   - Teach the content or concepts in a way that actively involved students in the learning process.

The next section shows how the three educators used these approaches in the fifth-grade inclusive classroom.

Using a Combined Approach to a Real-Life Statistical Problem—in a Real-Life Classroom

To introduce the lesson and the Lesson Organizer, the teacher proposed a puzzling question to the group: "How many fish are there in Canyon Lake?" (Any area lake that students are familiar with can be used.)

Building on Prior Knowledge and Gaining Prerequisite Skills

In small groups, the students brainstormed how they might propose solutions to the problem. Because it was impossible to physically count all the fish in Canyon Lake, the teacher introduced the students to the capture-recapture estimation method.

For example, she told them how the employees of the Parks and Wildlife Department would catch 120 fish, place tags on them, then release them back into the lake. Allowing time for the marked fish to mix with the unmarked fish, the employees would then catch a sample of 65 fish and find that only 2 of the fish they recaptured had tags. The employees would then write an equation to solve this proportion problem. They let N be the total number of fish in the lake. Because the ratio of marked fish in the population should be approximately equal to the ratio of marked fish in the sample, the students were directed to write the following equation:

$$\frac{\text{number of marked fish in the population}}{\text{total number of fish in the population } (N)} = \frac{\text{number of marked fish in the sample}}{\text{total number of fish in the sample}}$$

If we solve for N in the proportion:

$$\frac{120}{N} = \frac{2}{65}$$

$$7800 = 2N$$

$$3900 = N$$

Providing effective mathematics instruction for diverse and inclusive groups of students required systematic planning and powerful instruction.

The employees had estimated the total number of fish in the lake to be 3,900.

Following a Real-Life Simulation

It was now time to simulate this experiment with the class, using the Lesson Organizer Routine. First, the teacher directed the students to the lesson of the day and consolidated the goals. Next she quickly reviewed the prior lessons on constructing word problems with equations and fractions and reviewed the concept and examples of ratios. She then focused the students' attention to the paraphrase of the day's lesson and the steps involved, intermittently asking different students questions.

As the focus of the discussion was about estimating population proportions involving fish, she used Pepperidge Farm Goldfish™ for a task simulation, as follows:

- She divided the class into four cooperative learning groups and gave each group one bag of cheddar-flavored Goldfish and one bag of pretzel-flavored Goldfish. She directed the students in each group to first pour the bag of cheddar fish into a paper lunch

Lesson Organizer Routine

The Lesson Organizer Routine (Lenz et al., 1993) has three major components, the Lesson Organizer Device, the Linking Steps, and the Cue-Do-Review Sequence.

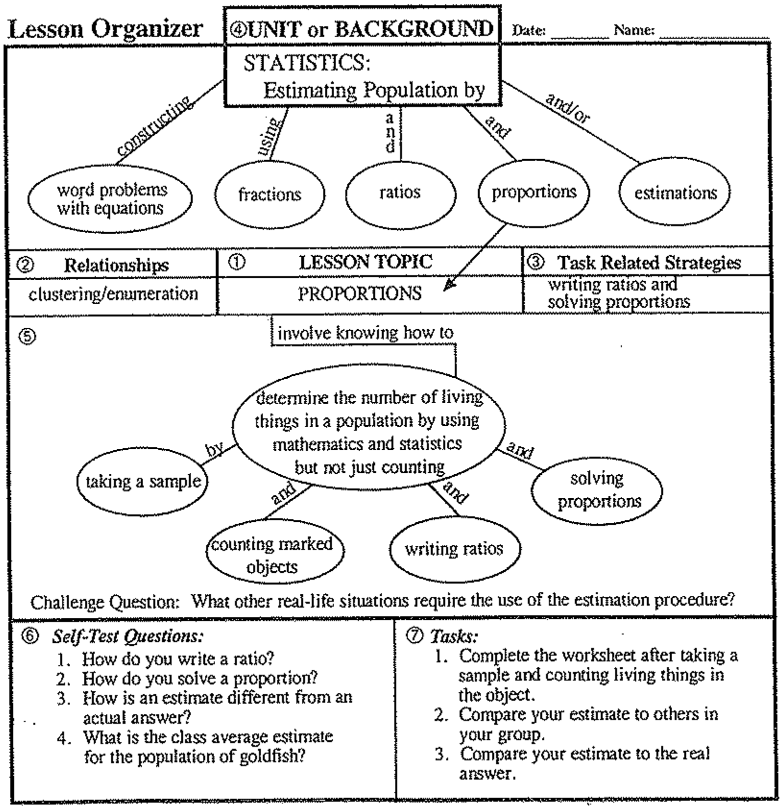
1. *Lesson Organizer Device.* The Lesson Organizer is a visual device teachers and students use together to focus students' attention on the important outcomes of a lesson, the relationships between abstract ideas and concrete objects and situations, and the relationship of content to students' background knowledge and experiences (see Figure 2 for a lesson on estimating populations; see Lenz et al., 1993, for more detailed instructions).

   Figure 2 shows how the students and the teacher used words and graphics that all students could easily understand, employing keywords, webs, ovals, lists, and arrows showing relationships. Moreover, the section labeled "Task-Related Strategies" identified strategies the students could use to gain, store, and express information as they worked to attain the goals of the lesson. The organizer also allowed the class to develop self-assessments ("Self-Test Questions") and required assignments and tasks (the section labeled "Tasks").
2. *Linking Steps.* The Linking Steps of the Lesson Organizer Routine interactively involve the whole class during the presentation of the Lesson Organizer. Using the acronym CRADLE as a mnemonic, the steps are as follows:
   C Consolidate Goals
   R Review Knowledge
   A Assemble a Visual Anchor
   D Describe and Map the Content
   L Link to Students' Lives
   E Explore Questions and Tasks.

   Using these steps, the teacher was able to provide students with the "big picture" of the lesson as they participated in its construction, thereby enhancing their learning.
3. *Cue-Do-Review.* The Cue-Do-Review Sequence of the routine addresses three important instructional phases of the lesson, as follows:
   - In the "Cue" phase, the teacher *cues* or informs the students about the lesson organizer.
   - In the "Do" phase, the teacher and students use the CRADLE steps to construct their own lesson.
   - During the "Review" phase, the students check and reinforce their understanding of the content and concepts covered during the lesson.

   In the fifth-grade classroom, the teacher found the Cue-Do-Review sequence very helpful with other lessons as well.

Figure 2: **Lesson Organizer**



Real data sets are authentic, interesting, and relevant; provide students with background information; and allow students to perform multiple analyses.

bag and then remove 20 fish from the bag.

- Next, she instructed the students to replace these 20 cheddar fish with 20 pretzel fish. She asked that one member of each group close the top of the bag and turn the bag over several times to mix up the fish.
- She then instructed each member of each group, one at a time, to take a handful of Goldfish out of the bag as a sample and count the total number of fish and the total number of pretzel fish in the handful. Each student was directed to record the total number in his or her sample and the total number of pretzel fish on his or her own worksheet, then replace the sample. The next group member would then take a sample, count, and record.
- After all the students had taken samples and recorded their findings, the teacher asked each student to calculate the total number of fish in the bag by writing a proportion to solve the problem. The teacher circulated around the room during the activity to make sure all the students were participating, and she provided assistance when it was needed.
- Next, she asked each group to discuss their answers. The teacher had each student write his or her total on the chalkboard, and the teacher drew a stem-and-leaf plot of the data. A stem-and-leaf plot is a quick method for looking at the distribution of the data. Actual numerical values are sorted, then displayed in two parts separated by a vertical line. The left part (stem) is usually the first digit and the right part (leaf) is the other digits.
- The teacher then instructed the students to calculate descriptive statistics for the population, including the mean, median, and mode.

After a discussion concerning the real answer, the teacher looked at the bag of the cheddar fish and read the number of fish that were actually in the bag. The class mean was found to approximate the real mean.

Powerful Instruction and Impressive Results

In reviewing student performances on a subsequent classroom test over the lesson and unit, the teacher noticed that as a group, the students had gained an average of at least 10 to 20 percentage points over previous test scores. She decided to continue to use the approach throughout the rest of the year with other units and lessons. The teacher also reported that the students' scores on the district's standardized competency-based test had improved. In May of the school year, she reported the results obtained by her class on the state mandated exam. Of the 21 students in the class, 7 students had been exempted from the exam by parent request; 1 student had been absent on the day of the exam so no score was available; 2 students did not attain passing scores; but 11 students, including 5 students with disabilities, had attained passing scores.

From this experience, the classroom teacher made several conclusions:

- She found that providing effective mathematics instruction for diverse and inclusive groups of students required systematic planning and powerful instruction.
- By combining approaches, she provided her students with better opportunities to learn because they were active in the learning process.
- She relearned the importance of getting students to connect new information to their prior knowledge and experiences.
- She found that collaboration and team planning enhanced her teaching style and promoted an openness to share ideas, concerns, and find solutions.
- The combination of approaches allowed her to present complex and abstract mathematical concepts in concrete and step-by step sequences.
- She found that her students had learned more as a group because the information was organized and structured for them, and the concepts she taught were clearly depicted.
- In using combined approaches, the teacher promoted both cooperative problem-solving and critical thinking among the students.

Finally, the fifth-grade teacher concluded that her students' learning and performance had improved because they were shown how to learn and how to complete their task and assignments—types of behavior that are critical to success in any classroom.

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