Promoting Strategic Math Performance Among Students with Learning Disabilities

Susan Peterson Miller and Sherri Strawser
University of Nevada, Las Vegas
Cecil D. Mercer
University of Florida

Throughout the eighties, much literature emerged documenting that students with learning disabilities (LD) have strategy deficits that negatively impact math performance. For example, several researchers reported that students with LD exhibit deficits in developing and executing specific math strategies and fail to use self-regulation (Fleischner & Garnett, 1980; Goldman, 1989; Pressley, Symons, Snyder, & Cariglia-Bull, 1989; Swanson & Rhine, 1985). Recent Investigations have identified some of the specific strategy problems that students with LD exhibit when attempting to solve math problems. For example, Montague, Bos, and Doucette (1991) found that although students with LD may possess a degree of strategy knowledge, such knowledge may be incomplete, insufficient, or inappropriate when applied to math word problems. These authors also found that students with LD did not paraphrase or visualize when attempting to solve word problems. Several other researchers (Hutchinson, 1993; Montague & Applegate, 1993; Zawaiza & Gerber, 1993) concurred with Montague et al.'s findings and further reported that students with LD lack critical strategies necessary for representing problems that involve converting linguistic and numerical information into math equations.

In spite of these deficits, individuals with LD can be taught specific strategies that will improve math performance. Fortunately, strategy instruction has been found to be effective for elementary students (Mercer & Miller, 1992a; Smith & Alley, 1981; Sugai & Smith, 1986; Willott, 1982); junior high students (Bennett, 1982; Case, Harris, & Graham, 1992; Montague, 1992; Montague, Applegate, & Marquard, 1993; Rivera & Smith, 1988); high school students (Hutchinson, 1993; Montague & Bos, 1986); and college students (Zawaiza & Gerber, 1993) with LD. So, regardless of age, teaching specific strategies for learning mathematics appears to provide students with the necessary tools to succeed.

The purpose of this article is to provide suggestions for classroom teachers who want to integrate strategy instruction into their math curriculum. Ideas for developing a strategic classroom environment are discussed in the first section, followed by an overview of several strategies designed to facilitate math achievement.

DEVELOPING A STRATEGIC CLASSROOM ENVIRONMENT

In addition to teaching specific strategies to students with LD, it is important that teachers create a classroom environment that promotes the use and maintenance of such strategies. Effective teacher behaviors greatly enhance the likelihood that students acquire and apply recently learned strategies to their math assignments. For example, teachers should involve students in goal-setting related to learning and using math strategies. Once these goals have been used to plan math instruction, several teacher behaviors will promote successful acquisition and retention of the specific math strategies taught. For instance, teachers can provide advance organizers, model strategic behavior and thinking, provide strategy practice, and reinforce strategy application through elaborated feedback. Each of these will help develop a strategic classroom environment.

Involve Students in Goal-Setting

Students with LD frequently have difficulty setting short- as well as long-term goals. They tend to be passive in their approach to learning and wait for teachers to tell them what to do and how to do it. Such an approach to academic tasks is likely to lead to feelings of dependency, inadequacy, and/or lack of interest. Thus, it is important that teachers seek ways to help students become more proactive and involved in planning their academic program. Certainly, one way to do so is to involve students in establishing math goals. Thus, elementary students with LD have been taught to set goals related to task completion of math seatwork (Schunk, 1985) while secondary students with LD have been taught to set computational fluency goals using computer-assisted instruction (Fuchs, Bahr, & Rieth, 1989). Both elementary and secondary students performed better on self-set rather than assigned goals, suggesting that goal ownership is important.

In addition to task completion and fluency goal-setting, students can become involved in determining which math strategies and skills will be taught and how much time will be spent learning them. Students can set short-term math goals that will help prepare them for long-term goals. The teacher's primary role in the goal-setting process is to make sure that the goals are high enough, yet realistic for the student to achieve. Figure 1 lists suggestions for implementing a goal-setting conference.

Figure 1. Goal-Setting Conference Suggestions

- Allow adequate time for the goal-setting conference (e.g., 10 to 15 minutes).
- Arrange to conference with students in a place that does not permit others to overhear.
- Encourage students to do most of the talking, that is, LISTEN!
- Begin the session by guiding students through a self-evaluation of their progress.
- Listen to students' ideas without interruptions or judgmental comments.
- If necessary, lead the students to revise their goals realistically by asking open-ended questions.
- Encourage students to set goals in skills at the acquisition, proficiency, and maintenance levels of learning. Students may refer to these goals as **easy, average**, and **difficult** goals.
- Record the results of the conference and provide copies for students. You may wish to develop a form for the conference that has space to record:

  the time frame for the goals
  self-evaluation of progress in "easy" skills
  self-evaluation of progress in "average" skills
  self-evaluation of progress in "difficult" skills
  amount of time spent working on each skill
  prerequisite skills that they did not have
  goals for "easy, average, and difficult" skills
  amount of time they plan to work on each skill

Provide Advance Organizers

Advance organizers provided at the beginning of a math lesson help students prepare for instruction. Typically, a verbal advance organizer tells the student what the lesson is going to be about, provides a rationale for learning the content, and ties the current lesson to previous learning. Advance organizers also can include prompts and cues for the student to use particular math strategies. Sample verbal advance organizers are presented in Table 1.

Advance organizers also can be presented in a written outline form, or they can involve asking students questions to elicit specific information about the upcoming lesson (Lenz, Alley, & Schumaker, 1987). Regardless of the format used, advance organizers help promote strategic behavior, enhance math content, and focus student attention.

Table 1. Sample Advance Organizers

"Remember yesterday when we learned to multiply using plates and bingo chips? Well, today we are going to learn how to multiply using plates and cubes. The problems will be the same type we did yesterday, so I know you'll be successful. Who can tell me about the adding-on strategy that we learned to save us time? . . . Good remembering! We'll use that strategy again today. Who remembers why it is important to learn to multiply? . . . Yes, those are good reasons."

"Yesterday we learned about pennies and nickels. Today we are going to learn about another coin called a dime. Remember, it is very important for us to learn about money because we have to use money to buy things that we want or need. When we go shopping we have to know what coins to give to the store clerk. Can anyone think of any other reasons that we need to know about money? . . ."

"Who remembers the trick we learned yesterday to help us remember when to regroup in subtraction? . . . Yes, we learned the BBB sentence. Bigger number on Bottom means Break down the ten and solve. Today we're going to learn how to use this sentence to help us subtract better. Just remembering the sentence won't help us unless we also know what the sentence means. So, let me show you what it means. . ."

Model Strategic Behavior and Thinking

Modeling is used frequently in mathematics instruction to show students how to perform a variety of algorithms. Specifically, the teacher writes a problem on the chalk board and then proceeds to solve it while the students watch. Modeling also can be used to show students how to use strategies that help them solve unknown problems. Modeling of this type involves showing students the strategy and letting them hear the related cognitive processes by "thinking aloud." In other words, the teacher demonstrates the steps to the strategy while verbalizing the related thinking. For example, a teacher showing students the counting-up strategy for subtraction might say, "To solve this problem, I'm going to read it first and try to remember the answer. 15 take away 9 . . . hmmm... I can't remember the answer so I'd better use my counting-up strategy. To use this strategy, I look at the bottom number, 9, and count up until I get to 15. I'll keep track of how many numbers I count to get to 15 by drawing tally marks. 10, 11, 12, 13, 14, 15. I drew six tallies, so my answer must be six. I think I'll do this one more time to be sure I'm right." This "thinking aloud" provides students with disabilities an opportunity to learn how to think strategically and mathematically at the same time. It provides the student with a method for determining unknown answers and reinforces the importance of self-checking for accuracy. Similar modeling procedures can be used for teaching a variety of math strategies.

Provide Strategy Practice

Integrating strategies into the math curriculum means that in addition to the time devoted to teaching specific math skills, teachers must allot time to teach specific strategies. Moreover, students will need time to learn and practice the strategy on a regular basis. Typically, math strategies involve several steps, which students must know at an automatic level before attempting to apply them to challenging math problems. At first, the strategies can be applied to easier math problems with teacher assistance, but gradually, teacher assistance is withdrawn and problem difficulty is increased. Ultimately, the student learns to discriminate when and how to use a variety of strategies. Although initially, it is natural to question whether time spent learning the strategy could be better spent practicing the math skill, in the long run, it becomes apparent that strategies promote students' success and independence as they solve various math problems.

Reinforce Strategy Application through Feedback

Once students have learned a new math strategy, it is important to reinforce the use of the strategy. Elaborated feedback routines are excellent opportunities for teachers to provide prompts or necessary reteaching when students fail to use a strategy and, consequently, make errors on their math work. Feedback routines are also useful for reinforcing students who used a strategy successfully and thus figured out some difficult problems.

To prepare for the elaborated feedback routine, the teacher first examines the students' math work. While noting error patterns, the teacher looks for evidence related to the presence or absence of strategy use. Once this is completed, the teacher meets with students individually. (In large classes the teacher may decide to meet with small groups of students who demonstrated similar errors.) The teacher begins the feedback routine by making at least one positive statement about the student's work, thinking pattern, or effort. Next, the teacher specifies error patterns in the student's work. Then, the teacher demonstrates how to complete the problem correctly using one of the strategies the student has been taught. The student then is given an opportunity to practice the strategy on a similar problem type. The teacher ends the feedback routine with a positive comment regarding the student's performance and sets positive expectations for future work on similar problems. Figure 2 illustrates this process.

Teachers who enhance their instruction with student-directed goal-setting, advance organizers, "thinking-aloud" modeling, adequate strategy practice, and elaborated feedback routines are creating a strategic classroom environment. These effective teacher practices promote strategy acquisition and retention and send a message to the students that learning how to solve math problems independently is important. This sets the stage for strategy instruction to occur successfully. The remainder of this article discusses several math strategies that have been particularly successful for students with LD.

Figure 2. Examples of Strategy Use and Feedback

- Make at least one positive statement about the student's work, thinking pattern, or effort.

  "I am very proud that you worked on your math for the whole independent work period today... Good job!"

- Specify the student's error pattern by focusing on the type or content of the error. Avoid using the word "you."

  "When zero is added to any number, the answer is the number."

  instead of

  "For most of these problems, you didn't remember the rule for adding zero."

- Demonstrate the correct completion of the problem using one of the strategies the student has been taught.

  "I will show you how to do these problems. Look at the first problem. It says '5 plus 0 is how many?' Because the first number is 5, I start by counting 5 tallies. Then, because the second number is 0, I don't count any more tallies, which is the same as counting nothing. The answer is all of the tallies I have counted. I have counted 5 so the answer to the problem, 5 plus 0, is 5."

- Give the student opportunity to practice.

  "Now please do the rest of the problems on the worksheet using the strategy."

- End with a positive comment about the student's performance and your expectations for future success with similar problems.

  "Good job adding 0 in all the problems. I am sure you will use the strategy for all the problems in which a 0 is added to other numbers."

DEVELOPING STRATEGIC LEARNERS

Teaching students to use specific math strategies helps them learn how to approach mathematical tasks in a logical manner. Although many strategies have been created for math instruction, three of the most common include seeking relevant relationships and rules, using mnemonic devices, and implementing self-monitoring procedures.

Relationships and Rules

When teaching mathematics to students with LD it is helpful to teach relationships and rules. Students can use relationships and rules to figure out challenging problems. For example, if a student knows how to add and understands the relationship between addition and subtraction facts, this

Figure 3. Examples of Mathematical Rules

RULES

ADDITION

- Any number plus zero is the number.
- Any number plus 1 is the next larger number.
- The order of numbers in an addition problem doesn't change the answer.

SUBTRACTION

- Any number take away zero is the number.
- Any number take away the same number is zero.
- Any number take away 1 is the next smaller number.
- In subtraction, when the **bottom** number in the ones column is **bigger** than the top number in the ones column, the **ten is *traded.*** (Bigger number on Bottom means Break down the ten and trade.)

MULTIPLICATION

- Any number times 0 equals 0.
- Any number times 1 equals the original number.
- 2 times any number equals the number added to itself.
- Changing the order of the numbers in multiplication does not change the answer.

DIVISION

- 0 divided by any number equals 0.
- Any number divided by 1 equals the number.
- Any number divided by the same number equals 1.

Figure 4. Examples of Mathematical Relationships

RELATIONSHIPS

ADDITION & SUBTRACTION

Addition and subtraction facts have the same numbers, but in different order.

$$\begin{array}{r} 9 \\ \underline{+4} \\ 13 \end{array} \qquad \begin{array}{r} 13 \\ \underline{-9} \\ 4 \end{array} \qquad \begin{array}{r} 13 \\ \underline{-4} \\ 9 \end{array}$$

Because addition and subtraction are related, you always can state a subtraction problem as an addition problem.

$$\begin{array}{r} 14 \\ \underline{-8} \\ \square \end{array} \qquad \begin{array}{r} \square \\ \underline{+8} \\ 14 \end{array}$$

To check a subtraction answer, add your answer to the number that is being subtracted. If the sum of these two numbers equals the top number in the subtraction problem, your answer is correct.

$$\begin{array}{r} 13 \\ \underline{-5} \\ 8 \end{array} \qquad \begin{array}{r} 8 \\ \underline{+5} \\ 13 \end{array}$$

MULTIPLICATION & DIVISION

Multiplication and division facts have the same numbers, just in different order.

$$7 \times 6 = 42$$
$$42 \div 6 = 7$$
$$42 \div 7 = 6$$

Because multiplication and division are related, you can always state division problems as multiplication problems.

$$56 \div 7 = \square$$
$$\square \times 7 = 56$$

To check a division answer, multiply your answer by the number that the total is being divided by. If the answer to this multiplication problem equals the total, your answer is correct.

$$24 \div 8 = 3$$
$$3 \times 8 = 24$$

information can be used to solve subtraction facts that are not yet committed to memory. Similarly, if a student understands the relationship between addition and multiplication, repeated addition can be used to figure out multiplication facts. Finally, if a student understands the relationship between multiplication and division, knowing multiplication facts can assist in determining the answer to division facts. Math rules, such as any number divided by itself is one, also can help students determine problem answers when their memory fails them.

However, teaching students to recognize and use math rules and relationships should not take the place of teaching concepts. Instead, rules and relationships should be taught along with the concepts. For example, students must understand that multiplication involves groups of objects and finding a total number, whereas division involves breaking a total number into number of objects per group or number of groups. Such understandings should precede memorizing facts and applying related rules and relationships. Figures 3 and 4 outline a variety of rules and relationships that students can use to help find solutions to unknown problems.

Mnemonic Strategies

Mnemonic strategies involve procedures that can be used to improve initial learning and later recall of important math information. Acronym mnemonics (i.e., forming a word from the initial letters of other words) used in mathematics can cue students to the steps involved in solving computation and word problems (Miller & Mercer, 1993). Research has shown

Table 2. Acronym Mnemonic Devices Designed to Facilitate Memory

DRAW *(Miller & Mercer, 1993)*	***SIGNS*** *(Watanabe, 1991)*
D iscover the sign.	**S** urvey question.
R ead the problem.	**I** dentify key words & labels.
A nswer or draw and check.	**G** raphically draw problem.
W rite the answer.	**N** ote operation(s) needed.
	S olve and check problem.
FAST DRAW *(Mercer & Miller, 1992B)*	***SOLVE*** *(Miller & Mercer, 1993)*
F ind what you're solving for.	**S** ee the sign.
A sk yourself, "What are the parts of the problem?"	**O** bserve and answer (if unable to answer, keep going).
S et up the numbers.	**L** ook and draw.
T ie down the sign.	**V** erify our answer.
(**DRAW** steps are the same as above).	**E** nter your answer.

that mnemonic memory devices are helpful for students with LD (Scruggs & Mastropieri, 1990; Watanabe, 1991). When teaching students to use acronym mnemonic strategies, it is important for students to learn the steps of the strategy to an automatic level. Thus, when they apply the steps to a math problem, they do not have to spend time trying to remember the acronym. It may be helpful for students to write the mnemonic steps on an index card to serve as a cue during initial attempts at using the strategy. Table 2 lists several acronym mnemonics that can be used for math instruction.

Similar strategies have been used whereby the student memorizes a nonsense acronym to help identify and recall a process to be followed when solving math problems. In such strategies, the acronym does not spell a word. For example RPV-HECC was used successfully to cue students to read, paraphrase, visualize, hypothesize, estimate, compute, and check when solving math word problems (Montague et al., 1993).

When teaching math strategies that involve mnemonic devices, it is important for the student to understand the vocabulary used in the steps. For some students, it is necessary to keep the vocabulary very simple and to limit the number of steps in the strategy. Low-level readers may need icons or pictures to help cue the various steps for solving the problem (Miller & Mercer, 1993).

Self-Monitoring Strategies

Another way to help students become more independent and successful in mathematics is to teach them self-monitoring strategies. A variety of strategies have been developed to help students regulate their math performance. For example, students have been taught self-recording, self-evaluation, and self-reinforcement procedures to increase their time on task and the accuracy of answers during independent practice of math skills. In one study (Hughs, Ruhl, & Peterson, 1988), students were taught to record the number of completed problems and the number of problems that were correct on a record-keeping form. They also were taught to evaluate their performance using a basic point system. The students reinforced themselves when their evaluation fell within a particular point range. These procedures increased accuracy and time on task for students with learning problems.

Another way to encourage self-monitoring in mathematics is to create checklists that cue students to use specific steps or strategies while solving math problems. The student refers to the list and checks off each step as it is completed. This helps students respond consistently and accurately to computation or problem-solving tasks (Miller, in press). Figure 5 illustrates a sample checklist.

As students become strategic learners, they learn to monitor their own math work. They learn to recognize errors or answers that do not make sense and then begin to self-correct their mistakes. Helping students achieve this level of performance should be a major goal for math teachers because this type of behavior greatly increases the likelihood that students will be able to apply their math learning beyond the classroom environment.

DISCUSSION

Although many students with LD have deficits in mathematics, it is encouraging to note that promoting strategic math performance can have a positive effect on their achievement. Teachers who decide to integrate strategy instruction into their math curriculum should focus on two things. First, it is important to create a strategic classroom environment where students are encouraged to develop strategic thinking. Second, students must be taught explicit strategies to apply to their particular math curriculum. Both of these components of strategic math instruction require extra planning and instructional time, but time spent in this manner is a wise investment in the future math success of students with LD. Moreover, time is saved in the long run because students are able to perform independently and require less reteaching of critical skills.

The ideas and suggestions presented in this article are designed to get teachers started including strategies in their math instruction. It should be noted, however, that other strategies exist that may also be appropriate for particular students. Educators are, therefore, encouraged to continue their exploration of strategies that will enhance their effectiveness as teachers. Moreover, it is hoped that teachers will create their own strategies and ultimately encourage their students to create strategies as mathematical needs emerge.

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Figure 5. Sample Self-Monitoring Checklist for Solving Word Problems

My Word Problem Checklist

- ☐ Read the problem.
- ☐ What is the question the problem asks?
- ☐ To answer the question, do I have to:
  - ☐ ADD
  - ☐ SUBTRACT
  - ☐ MULTIPLY
  - ☐ DIVIDE
- ☐ What information is not needed?
- ☐ Write out the problem using numbers.
- ☐ Solve the problem.
- ☐ Check the answer.

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