Education and Training in Autism and Developmental Disabilities, 2017, 52(3), 317–331

Concrete and App-Based Manipulatives to Support Students with Disabilities with Subtraction

Emily C. Bouck, Courtney Chamberlain, and Jiyoon Park
Michigan State University

Abstract: Manipulatives support students with and without disabilities in mathematics. However, as students age, concrete manipulatives can be limiting and potentially not age appropriate (Satsangi, 2015). An alternative is virtual manipulatives, including app-based manipulatives. This study compared the use of app-based manipulatives to concrete manipulatives in supporting students with disabilities in solving subtraction problems with regrouping. Using an adapted alternating treatment design with three middle school students with disabilities, the researcher found app base 10 blocks were more effective in terms of solving subtraction with regrouping for two of the students. They also found that all three students were more independent with the app-based manipulatives, although only two of the three students preferred the app-based manipulatives to the concrete manipulatives.

Students with disabilities struggle with mathematics, and generally more so than students without disabilities (The Nation's Report Card, 2016). The average score for fourth-grade students with disabilities on the 2015 National Education Assessment Program (NAEP) was 218, as compared to the average score of 244 for fourth-grade students without disabilities. Similarly, for eighth-grade students with disabilities, the 2015 NAEP data suggested an average score of 247 for students with disabilities as compared to 287 for students without disabilities (The Nation's Report Card, 2016).

Given the struggle students with disabilities face in mathematics, researchers and practitioners have sought effective practices. In mathematics, manipulatives are considered an effective strategy for teaching students with and without disabilities (Marley & Carbonneau, 2014). Mathematics manipulatives – generally assumed to be concrete manipulatives – are physical objects students can manipulate with their hands. Over the past few decades, an alternative form of mathematics manipulatives was developed and used to support students: virtual manipulatives. Virtual manipulatives are digital manipulatives that serve similar functions as concrete manipulatives – and are often similar to concrete manipulatives – but exist in a digital form (Bouck & Flanagan, 2010). Previously, virtual manipulatives were online (or Internet-based) manipulatives, such as ones available from the National Library of Virtual Manipulatives (NLVM). However, more recently, with the increase in attention to mobile devices, a newer form of virtual manipulatives exists: app-based manipulatives.

Although the use of concrete manipulatives is supported in research and practice for students with disabilities (Lai & Berkeley, 2012; Maccini & Gagnon, 2000), concrete manipulatives posses limitations. Hence, there is merit to considering virtual manipulatives – both online and app-based – for students with disabilities. For one, virtual manipulatives may be more age-appropriate for secondary students with disabilities (Satsangi, 2015). The use of concrete manipulatives typically designed for younger students (e.g., base 10 blocks) can be stigmatizing or embarrassing to use for secondary students. Virtual manipulatives may also reduce the cognitive load for students with disabilities (Suh & Moyer, 2008). Given the built-in supports or constraints within virtual manipulatives, students

Correspondence concerning this article should be addressed to Emily C. Bouck, 349A Erickson Hall, 620 Farm Lane, Michigan State University, East Lansing, MI 48824. E-mail: ecb@msu.edu

may experience a decreased cognitive load when using them (Moyer, Niezgoda, & Stanley, 2005; Suh & Moyer, 2008).

Previous Manipulatives Research

Prior research, albeit limited, exists on both the use of virtual manipulatives for students with disabilities and the comparison of concrete and virtual manipulatives for students with disabilities. In a multiple baseline single subject design study involving three high school students with learning disabilities, Satsangi and Bouck (2015) explored the use of online polynominoes (i.e., digital tiles) from the NLVM. The students successfully used the online manipulatives to solve area and perimeter problems as well as maintain and generalize these skills. Bouck, Flanagan, and Bouck (2015) also explored the use of online manipulatives to support middle school students with disabilities in solving area and perimeter problems. Also using the polynominoes from the NLVM, Bouck et al. (2015) conducted a pretest-posttest study and found that students answered more area and perimeter questions correctly – and attempted more problems – on the posttest following instruction with online manipulatives.

For comparison research, Bouck, Satsangi, Doughty, and Courtney (2014) compared concrete base 10 blocks to online base 10 blocks from the NLVM to support single-digit or double-digit subtraction with regrouping for three elementary students with autism. Bouck et al. (2014) found online and concrete manipulatives supported students correctly solving subtraction problems, although they were slightly more independent in completing the task analysis steps with the online manipulatives. The students also expressed a preference for virtual manipulatives. Finally, Satsangi, Bouck, Taber-Doughty, Bofferding, & Roberts (2016) compared the use of a online algebraic balance scale from the NLVM to a concrete algebraic balance scale in an alternating treatment design with three high school students with learning disabilities. Satsangi et al. (2016) found that the three students were successful with both types of manipulatives in solving linear algebra equations, although the students preferred the virtual manipulatives.

The existing research to date focuses on online manipulatives, such as from the NLVM. To date, very limited research exists on app-based manipulatives to support students with disabilities. The limited research is, of course, likely attributed to the relative recency of app-based manipulatives. Although not a manipulative app, recent research by Bryant et al. (2015) suggested no differences across three conditions were found in terms of students acquiring multiplication facts when comparing their learning via a math app, teacher-directed instruction, or a combination of app and teacher-directed instruction. Similarly, while not an app-based manipulative, Ok and Bryant (2015) found fifth-grade students with learning disabilities improved in their multiplication facts following an intervention of an app focused on multiplication fact practice.

While online manipulatives – such as the NLVM, provide educators with free Internet-based virtual manipulatives to support students across grades in multiple mathematical areas, online manipulatives require Internet access and do not work on tablets, such as iPads, given they often use JAVA. Given schools' increasing use of mobile devices, such as iPads (Pilgrim, Bledsoe, & Riley, 2012), it is important to consider how app-based manipulatives compare to concrete manipulatives in supporting students in mathematics and if app-based manipulatives are a viable option for students with disabilities. The research questions for this study include: (a) what number of subtraction mathematics problems do students solve accurately when using app-based manipulatives and concrete manipulatives?; (b) what percentage of subtraction mathematics problems do students solve independently when using app-based manipulatives and concrete manipulatives?; and (c) what are student preferences when considering app-based or concrete manipulatives?

Method

Participants

Three middle students participated in the study. Each received special education services from their school district via a pullout program (i.e., all core content courses taught by a special education teacher, including mathematics). Participants were chosen according to the following: (a) teacher recommendation for students struggling with double-digit or triple-digit addition or subtraction; (b) confirmation of struggles with double- or triple-digit addition or subtraction through independent KeyMath™-3 Addition and Subtraction subtest with ceilings in double- or triple-digit addition and subtraction, and (c) fine motor ability to move concrete manipulative blocks and navigate a touch-based iPad application.

José. José was a 14-year-old Hispanic student in eighth grade. José was a pleasant young man who played on his school's basketball team and strongly disliked math. His special education eligibility was in the area of mild intellectual disability. According to his performance on the *Wechsler Intelligence Scale for Children-Fourth Edition* (WISC-IV), José's full-scale IQ was 62. His performance on the mathematics subtests of the *Wechsler Individual Achievement Test-Third Edition* (WIAT-III) indicated his Math Problem Solving standard score of 71 and Numerical Operations standard score of 66 were in the below average and low ranges of performance, respectively. Additional mathematics scores from the KeyMath™-3 Diagnostic Assessment and STAR Math Assessment suggested grade equivalent mathematics performance at the second grade level (2.9 and 2.1, respectively). José's teacher indicated his mathematics performance was inconsistent, especially with regards to triple-digit subtraction.

Ellen. Ellen was a 13-year-old Caucasian student in seventh grade. Ellen was a friendly girl who enjoyed drawing and coloring. Her special education eligibility was in the area of mild intellectual disability. According to her performance on the WISC-IV, Ellen's full-scale IQ was 68. Her standard score of 62 on the *Woodcock Johnson Tests of Achievement-Third Edition* (WJ-III-Ach) Broad Math composite was in the very low range of performance. Her score on the STAR Math Assessment suggested grade equivalent mathematics performance at the third grade level (3.2). Ellen also had mental health issues and took medication for bipolar disorder. She had inconsistent attendance and often complained of not feeling well.

Vince. Vince was an 11-year-old Hispanic male in sixth grade. Vince was very quiet and reserved but always prepared for class and for sessions. His special education eligibility was in the area of specific learning disability for reading comprehension and mathematics calculation. According to his performance on the *Woodcock-Johnson Tests of Cognitive Abilities-Third Edition* (WJ-III-Cog), Vince's Global Intellectual Ability score was 84. His standard score of 79 on the Applied Problems subtest and 73 on the Calculation subtest of the WJ-III-Ach was in the low range of performance. Additionally, his score on the KeyMath™-3 Diagnostic Assessment suggested grade equivalent mathematics performance at the second grade level (2.2), while his score on the STAR Math Assessment suggested grade equivalent performance at the third grade level (3.3). Vince struggled with processing, as evident by both researcher observation as well as stated by his special education teacher. When asked a question or tasked with solving a problem with the manipulatives, he was very slow to respond. His teacher also discussed his processing struggles and his struggle to apply concepts repeatedly presented.

Setting

Study sessions occurred at a public middle school in a rural Midwest town. At the time of data collection, the school enrolled approximately 712 students in sixth through eighth grade. Approximately 86% of the student population were Caucasian, 12% Hispanic, 1% Multiracial, and less than 1% were African American and Alaska Native/American Indian. Approximately 16% of the student body were identified as students with disabilities. Data collection occurred in a small room in the building's administrative area as well as occasionally unoccupied areas of the school library. Each space had at least three chairs and tables large enough for the students to use the concrete manipulative blocks or iPad as well as write their responses down on probes. Sessions were conducted in an one-on-one environment between a researcher (one of the authors) who implemented the study and the student; sessions in which inter-observer agreement data were collected had a second researcher present.

Materials

Materials included pencils, researcher-constructed probe sheets, concrete manipulative blocks and place value sheets, and an iPad with the base 10 blocks app. The researchers gave students a pencil in each session and a probe sheet containing five subtraction problems with regrouping, with each problem presented vertically. José and Vince completed assessments with triple-digit subtraction problems while Ellen double-digit subtraction problems, selected based on students' instructional mathematics level and performance on the KeyMath™-3. Note, each five-question problem set was unique. The researchers developed all double-digit and triple-digit subtraction problems needed across the study and then randomly selected problems for each problem set; each problem set was randomly selected for its use (i.e., phase of the study).

During the concrete manipulative blocks phase, students were provided with a place value sheet and the concrete base 10 blocks. The place value sheet contained three columns with pictorial representations of manipulative blocks at the top of each column. The right column was labeled "Ones" with a ones block at top, the middle column was labeled "Tens" with a tens block at top, and the left column was labeled "Hundreds" with a hundreds block at top. Two sheets were provided, one for the minuend and one for the subtrahend; the place value sheets were also consistent with the virtual manipulatives app. Researchers provided two different colored base 10 blocks; students used one color for the minuend and the other for the subtrahend. Researchers provided more than enough ones, tens, and hundreds blocks – as applicable – at the start of each session to the left side of the place value sheets. Students set up blocks on the place value sheets to represent each number of the problem, regrouping tens or hundreds blocks for ones or tens when regrouping was required to solve the problem, and removing blocks when subtracting.

During the app-based manipulative blocks phase, students were provided with an iPad on which a manipulative app was downloaded. The app, Base 10 Blocks Manipulative Version 1.1.0, was developed by Brainingcamp, LLC and available for a $0.99 purchase on iTunes. The researchers selected this app after evaluating multiple base 10 block app for iOS devices. Although the app has limitations, this app possesses many positives to make it an appropriate choice. For one, the blocks on the app – and how they operate – are very similar to concrete base 10 blocks. The app was also flexible to support students in place value, addition, and subtraction, and worked with multiple place values, including decimals. Finally, the app from Brainingcamp was intuitive to use but also had built-in constraints beyond what concrete manipulatives could do (i.e., it would not let students regroup from the subtrahend rather than the minuend).

Depending on the type of subtraction problem, the app presented two to three columns and a separate row for both the minuend and the subtrahend. As with the place value sheets in the concrete manipulative blocks phase, the top of the columns contained pictorial representations of ones, tens, and hundreds, if applicable, blocks. Students set up blocks by touching the pictorial block representations and dragging them from the tops of the columns to the respective rows for the minuend and the subtrahend. To ungroup a hundreds or tens blocks, students moved the hundreds blocks to the tens place and/or tens blocks to the ones place. When this was done, the block being moved ungrouped itself into smaller units (i.e., ten tens blocks or ten ones blocks). To subtract, students dragged blocks from the subtrahend row to same-unit blocks in the minuend row. When two same-unit blocks from each number came into contact, they became semi-transparent to signify subtraction (i.e., as if the student had removed the blocks from the place value chart). Of note, the iPad app displayed the problem – and the answer – on the app at the bottom of the screen. Researchers covered the problem and answer with small sticky notes; no student ever tried to remove the sticky notes from the iPad or questioned researchers about it.

Independent and Dependent Variables

The independent variable for the study was use of manipulatives; students used concrete manipulative blocks or app-based manipulative blocks to complete subtraction problems. Using concrete manipulative was defined as setting up and moving the base 10 blocks to solve each subtraction problem. Using an app-based manipulative was defined as setting up and dragging the digital base 10 blocks in the app to solve each subtraction problem.

The dependent variables for the study included (a) the number of subtraction problems the student answered correctly out of five (i.e., accuracy), (b) the amount of time it took students to complete each assessment (i.e., task completion time), and (c) the percentage of subtraction task analysis steps they completed independently without prompting (i.e., independence). The researchers used event recording for accuracy and independence and duration for task completion time. Accuracy was calculated by summing the number of problems a student answered correctly on a probe out of five. Task completion time was measured by the amount of time it took a student to solve the five problems, with or without manipulatives; the timer was started when the student received the subtraction probe sheet and ended when the student wrote the last answer on the probe. Independence was calculated by determining the number of task analysis steps the student completed independently in each session and dividing that by the total number of steps (concrete, app-based, and no manipulative task analysis recording sheets available upon request for double-digit and triple-digit subtraction). There were 14 steps for the triple-digit subtraction problems with both manipulative (total of 70 across each probe), and 9 steps for double-digit subtraction problems with manipulative, for a total of 45.

Experimental Design

An adapted alternating treatment design was employed, with four phases: baseline, intervention, best treatment, and generalization (Sindelar, Rosenberg, & Wilson, 1985; Wolery, Gast, & Hammond, 2010). In the present study, students solved subtraction problems across three alternating conditions during the intervention phase: concrete manipulatives, app-based manipulatives, and extended baseline in which no manipulatives were used. Five sessions of each condition were alternated at random, with no more than two sessions of the same condition (concrete, app, or no manipulative) in a row. Using this design, the authors were able to determine the effectiveness of each type of manipulative on the students' subtraction problem solving and create an experimental control for each student (Sindelar et al., 1985). Across all sessions, the researchers served as the implementers; the teacher was not involved with the implementation of the study procedures and only participated in social validity.

Procedures

Baseline. For baseline, students were required to complete subtraction assessments at their mathematics level (i.e., triple-digit or double-digit subtraction with regrouping) with no manipulatives. Each subtraction probe was a sheet of paper with five subtraction problems presented in two columns on one side of the sheet, and each problem was presented vertically. Students were provided with a pencil and asked to solve the five problems. Researchers were prepared to offer prompting if students failed to initiate solving within 10 seconds.

Pre-training. Before intervention, students were provided training on both the concrete manipulatives and app-based manipulatives. To train students on each type of manipulative, the researchers employed explicit instruction (Doabler & Fien, 2013). With the explicit instruction, the researchers first demonstrated and used think-alouds for how to use the manipulatives to solve practice subtraction problems at their mathematics level (i.e., double- or triple-digit subtraction problems with regrouping); the modeling portion for the explicit instruction was done for two problems each session. Consistent with explicit instruction, after two sessions of modeling (i.e., demonstrating and use think-alouds), the students worked to solve two problems, and researchers provided prompts and cues as needed. Finally, after two sessions with guiding, the students completed five problems independently. If students correctly solved 80% of the problems in the independent phase with each manipulative, they were considered successfully trained with that manipulative. Each student had to score 80% during one training probe for each manipulative type to move into the intervention phase. Ellen was trained in one session for each manipulative condition. José required two sessions for each manipulative condition. Vince was trained after three separate sessions for both the concrete and app-based manipulatives.

During the concrete manipulatives explicit instruction, students were trained to read the subtraction problem on the probe sheet and set up the hundreds blocks (triple-digit subtraction only), tens blocks, and ones blocks for the minuend in the first row of the place value sheet using one of the two sets of different color concrete manipulative blocks. These steps were repeated for subtrahend using the remaining set of concrete manipulative blocks, and students were trained to place blocks for the subtrahend in the second row of the place value sheet. If the minuend ones were smaller than the subtrahend ones, students were trained to regroup a tens block from the minuend for 10 ones blocks. The ten ones blocks were then added to the ones of the minuend. Students solving triple-digit problems were also trained to regroup a hundreds block from the minuend into 10 tens blocks should the minuend tens be smaller than those in the subtrahend. The 10 tens blocks were then added to the tens of the minuend. Finally, students were trained to subtract subtrahend blocks from same-unit minuend blocks and write the answer on the probe.

Similarly, students were trained to solve subtraction problems using the app-based manipulatives. As with the place value sheets for the concrete manipulatives, the app displayed two rows in which students were trained to drag ones blocks, tens blocks, and hundreds blocks (for triple-digit subtraction only) to set up the minuend and subtrahend subtraction problem numbers. Minuend blocks were green, while subtrahend ones were red. If students accidentally selected too many blocks, they dragged the extra blocks to a small trash bin icon in the app to delete. Next, students were trained to regroup tens and hundreds blocks in the minuend by dragging them to the column to the right (i.e., the ones and tens column, respectively). Upon doing so, the blocks ungrouped into smaller units (e.g., 10 ones blocks from the ten's block and 10 tens blocks from hundred's block). To subtract, students were trained to drag blocks from the subtrahend to same-unit blocks in the minuend. When paired with a same-unit block, the paired blocks became semi-transparent, indicating they were subtracted. Once all the blocks from the subtrahend were paired with blocks from the minuend, or subtracted, they were trained to count the remaining opaque blocks and write the answer on the probe.

Intervention. Following trainings, students completed the intervention phase, alternating between concrete manipulative blocks, app-based manipulative blocks, and no manipulatives (i.e., extended baseline) to solve subtraction problems with regrouping. Each condition included five sessions, with five subtraction problems per session. Session order was randomly determined with no more than two of the same conditions occurring consecutively. The system of least prompts was used during the intervention phase to assist students if they did not engage in the correct task analysis step within 10 seconds of completing the previous step (i.e., system of least prompts delivered after a 10-second time delay). The researchers used prompts that ranged from less intense visual (i.e., gesturing) and verbal prompts (i.e., indirect verbal prompts such as, "What comes next?") to more intense verbal prompts (i.e., direct verbal prompts indicating the next step), modeling, and partial physical assistance (Doyle, Wolery, Ault, & Gast, 1988). More intensive prompts were given when students did not respond to previous prompts.

Concrete manipulative blocks. During the concrete manipulative intervention condition, students were provided with two place value sheets, two different colored sets of concrete base 10 blocks, a probe sheet with five subtraction questions, and a pencil. The two sets of blocks were placed on the table for students to select from for the subtraction problem minuend and subtrahend. Upon reading the subtraction problem on the probe sheet, students were expected to set up the correct number of blocks in each cell of the place value sheets. They also regrouped larger blocks for smaller ones in the minuend in order to subtract the subtrahend from the minuend. Next, they counted the remaining blocks and wrote the answer on the probe sheet.

App-based manipulative blocks. In this condition, students used the Base 10 Blocks Manipulative iPad app (Brainingcamp, LLC, 2015) to solve subtraction problems. In addition to the iPad, students were provided with a probe sheet with five subtraction problems and a pencil. The iPad was opened to the app, and researchers adjusted the app settings prior to the session to fit students' mathematics levels. The app for Ellen had two rows and columns to represent double-digit subtraction, while the app for José and Vince had two rows with three columns to represent triple-digit subtraction. Students touched and dragged blocks down from representative pictures at the top of each column to represent the problem. After the blocks were set up, students regrouped and subtracted. Next, students counted the remaining blocks and wrote the corresponding answer on the probe sheet. As noted previously, sticky notes were placed over the problem and answer at the bottom of the iPad display so students could not see them.

No manipulative. The baseline condition of no manipulatives was extended into the intervention phase to serve as a control for student performance to determine a functional relation. Researchers provided students with a five problem probe sheet and a pencil. Students solved the problems without any aids (i.e., concrete or app-based manipulatives).

Best treatment. Following the intervention phase, students completed three sessions using the intervention condition – concrete or app-based manipulatives – most effective for them during intervention. To determine best treatment, researchers used percentage of non-overlapping data (PND; Gast & Spriggs, 2010); PND is a commonly used method within alternating treatment designs to compare the data from one condition to another (Gast & Spriggs, 2010). Note, PND was not used an effect size for accuracy; the researchers used Tau-U for calculate accuracy effect size, as will be explained later. Specifically, the researchers determined PND first for the accuracy, and then, if there was no difference, for the independence. To calculate PND, the researchers found the number of sessions that one condition (e.g., app-based manipulatives) was more effective than the other (e.g., concrete manipulatives) in the target dependent variable – first by accuracy and then independence, if needed. They then divided the summed number by five and multiplied by 100 (Wolery, Gast, & Hammond, 2010). For Ellen, best treatment was calculated based on independence data as both manipulative conditions were equally effective for her, while for José and Vince, best treatment was calculated based on accuracy data. For all three students, the best treatment condition corresponded to their preferred condition.

Similar to the intervention phase, students were provided with a probe sheet with five problems and a pencil in addition to the concrete manipulative blocks or iPad with the app, depending on the condition most effective for him or her. Best treatment followed the same procedures as intervention and included the same data collection (i.e., accuracy, task completion time, and independence). The system of least prompts was also used when students did not initiate a step or the correct step within 10 seconds of completing the previous one.

Generalization. Two generalization sessions were implemented following best treatment to evaluate for any generalization of the subtraction skills. Consistent with baseline and extended baseline, students were given a five-problem probe sheet and a pencil. No aids in the form of concrete or app-based manipulative blocks were provided to assist students with problem solving.

Inter-Observer Agreement and Treatment Integrity

Inter-observer agreement (IOA) data and treatment integrity data were recorded for both the independent variable (manipulative condition) and prompts (i.e., independence data). IOA data were recorded for each student for (a) two sessions (40%) for each intervention condition (concrete manipulative, app, or no manipulatives), (b) at least 40% of baseline sessions (two for Ellen and Vince and three for José), (c) 33% for the best treatment sessions, and (d) 50% of the generalization sessions. During sessions in which IOA data were collected, two researchers were present and collected data. IOA was calculated by summing the number of agreements for both dependent variables examined – accuracy and independence – and dividing it by the number of agreements plus disagreements. IOA was 100% for accuracy for each student for each phase and condition. IOA for independence was 100% for all three students during baseline, extended baseline during intervention, and generalization. It was also 100% for José and Ellen for both intervention conditions and best treatment; IOA for Vince was 97.2% for the app intervention condition and best treatment and 92.9% for the concrete manipulatives condition.

Treatment integrity data were recorded across 40% of intervention and 33% of generalization sessions. The integrity data monitored included whether the students were provided a probe sheet, the appropriate type of manipulatives, and researchers implemented the system of least prompts. Treatment fidelity for each student for each condition and phase was 100%.

Social Validity

Social validity interviews were conducted after intervention with the teacher and the students. Students were asked questions regarding their perceptions about each type of manipulatives, including which type they preferred. The students' special education teacher was interviewed about her mathematics instructional practices, her students' mathematics abilities, and her opinion of using each type of manipulative during instruction.

Data Analysis

To analyze the data, researchers conducted visual analysis, including calculating level, trend, and effect sizes (Gast & Spriggs, 2010). Researchers calculated level by finding the stability of the data in each phase or condition. To do this, researchers first calculated the median for each dependent variable for each student, and then a 20% interval around each median, referred to as stability envelope (e.g., if the median was 4, the stability envelope range would be 3.2–4.8). Data were stable if 80% of a student's data fell within 20% of the median for each dependent variable analyzed (Gast & Spriggs, 2010). Researchers used the split-middle method to determine trend (White & Haring, 1980). Researchers split the data for each phase and condition in half and calculated the mid-rate and mid-date. Next, they drew a line between the mid-rate and mid-date and determined if the line was accelerating, decelerating, or zero-celerating (Gast & Spriggs, 2010). The researchers used Tau-U to calculate effect size for the accuracy data. Tau-U contrasts each intervention condition with baseline (Parker, Vannest, Davis, & Sauber, 2011). To calculate the Tau-U, researchers used the web-based calculator (see http://www.singlecaseresearch.org/calculators/tau-u; Vannest, Parker, & Gonen, 2011). Tau-U scores less than or equal to 65% suggest a small effect, 66–92% a medium effect, and 93% and above a large effect (Parker, Vannest, & Brown, 2009).

Results

José

José struggled with triple digit subtraction with regrouping. José's accuracy with the app base 10 blocks was superior to that with the concrete base 10 blocks with a PND of 40% (concrete to app PND = 20%). José also indicated his preferred treatment condition was the app.

José's accuracy data. During baseline, José's average number of problems correct was 2, with a range from 0–4 (see Table 1 & Figure 1). José's inconsistent performance during baseline was consistent with teacher reports. During intervention, he averaged 3.8 and 4.0 problems correct in the concrete and app conditions, respectively. Compared to baseline data, José's Tau-*U* for accuracy was 69% for concrete and 83% for app-based manipulatives – both a medium to high effect. José's extended baseline showed slight improvement from baseline ($\mu = 2.4$ vs. $\mu = 2$), although he answered more correctly, on average, during generalization (3.5). José's average score during best treatment was higher than during his intervention condition ($\mu = 4.3$ vs. $\mu = 4$).

TABLE 1

Participant Accuracy, Independence, and Task Complete Time Data Across Phases

				Intervention				
Participant	*DV*	*Data*	*Baseline*	*Concrete Manipulative*	*App-based Manipulative*	*No Manipulative*	*Best Treatment*	*Generalization*
José	Accuracy	*u*	2	3.8	4	2.4	4.3	3.5
		level	variable	variable	variable	variable	variable	
		trend	decelerating	accelerating	accelerating	accelerating	zero-celerating	
	Independence	*u*	100%	97.5%	98.2%	100%	100%	100
		level	stable	stable	stable	stable	stable	
		trend	zero-celerating	decelerating	decelerating	zero-celerating	zero-celerating	
	Time	*u*	3.4	13.5	8.7	2.9	7.8	3.3
		level	stable	stable	stable	stable	stable	
		trend	decelerating	decelerating	accelerating	decelerating	decelerating	
Ellen	Accuracy	*u*	0.4	5	5	1.8	5	5
		level	stable	stable	stable	variable	stable	
		trend	accelerating	zero-celerating	zero-celerating	accelerating	zero-celerating	
	Independence	*u*	100%	98.2%	99.5%	100%	100%	100
		level	stable	stable	stable	stable	stable	
		trend	zero-celerating	accelerating	zero-celerating	zero-celerating	zero-celerating	
	Time	*u*	2.7	11.1	8.2	2.4	7.2	2.3
		level	variable	variable	stable	variable	variable	
		trend	decelerating	decelerating	accelerating	accelerating	zero-celerating	
Vince	Accuracy	*u*	0.4	4.2	4	0	4.7	0
		level	variable	stable	variable	stable	stable	
		trend	decelerating	accelerating	decelerating	zero-celerating	accelerating	
	Independence	*u*	100%	88.0%	91.5%	100%	94.8%	100
		level	stable	stable	stable	stable	stable	
		trend	zero-celerating	decelerating	accelerating	zero-celerating	accelerating	
	Time	*u*	2.8	23.4	19.6	3.2	21.7	2.5
		level	variable	stable	variable	variable	stable	
		trend	decelerating	decelerating	decelerating	accelerating	decelerating	

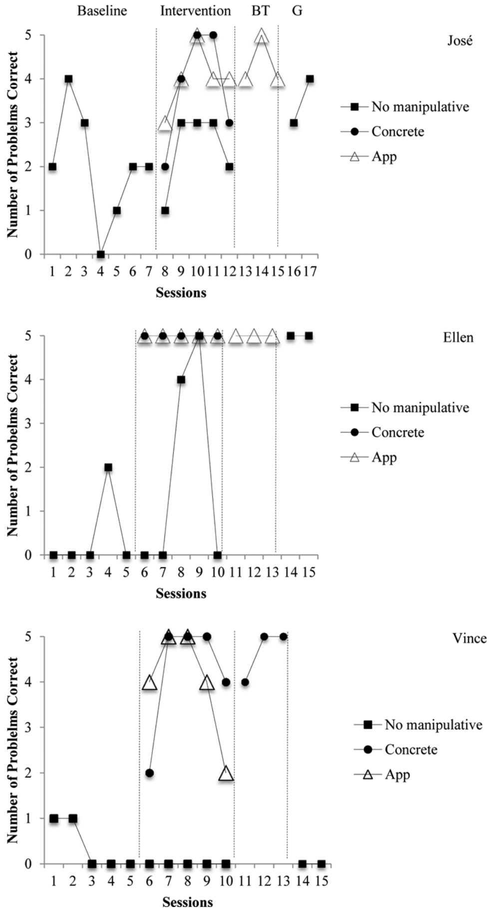

Figure 1. Accuracy of Subtraction Probes.

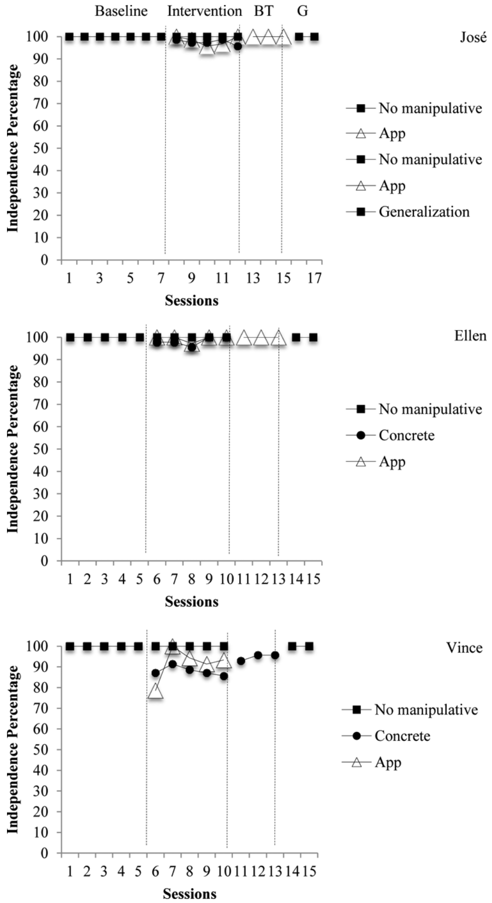

Figure 2. Independence in Completing Subtraction Problems Task Analysis Steps.

José's independence data. José was very independent during all phases of the study, but was 100% independent during baseline, extended baseline (no manipulative condition during the intervention phase), best treatment, and generalization. Across the intervention conditions, José needed few prompts, with an average of 97.5% independence across the task analysis steps in the concrete manipulative condition and 98.2% independence across the task analysis steps in the app-based manipulative condition (see Figure 2). When José needed prompts, as few as they were, they were gestures and usually involving steps related to regrouping. José proceeded through steps so quickly at times that he would just skip over a step in an effort to finish quicker.

José's task completion time data. José completed the five problems during baseline, extended baseline, and generalization phases in a relatively short amount of time – an average of 3.4 minutes, 2.9 minutes, and 3.3 minutes, respectively (see Figure 3). In terms of efficiency, José spent less than one minute per problem during baseline, extended baseline, and generalization. José completed the five problems in a shorter amount of time in the app-based manipulative condition (μ = 13.5 minutes) as compared to the concrete manipulative condition (μ = 8.7 minutes). The difference in efficiency (minutes spent per problem) when comparing the concrete and app-based averages was about one minute (2.7 minutes per problem to 1.7 minutes per prob-

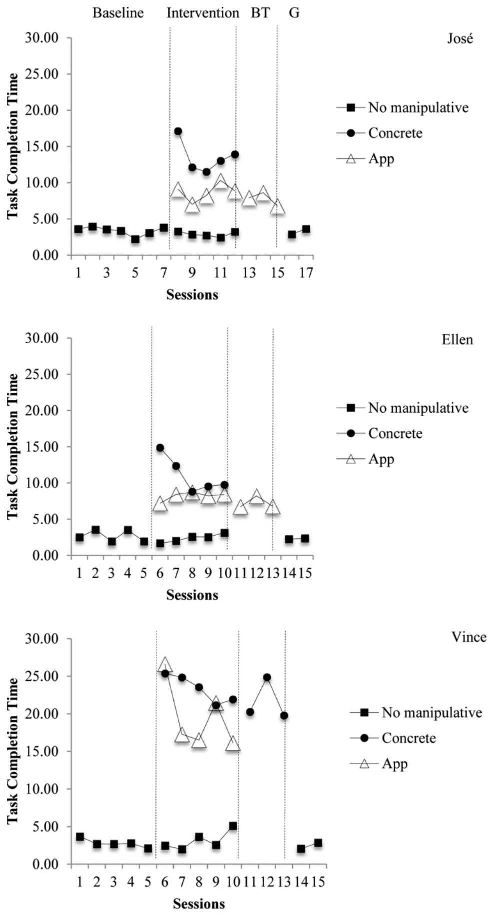

Figure 3. Task Completion Time Per Subtraction Probe.

lem). José decreased his task completion rate during best treatment (μ = 7.8 minutes).

Ellen

Ellen's data indicated difficulty with double-digit subtraction with regrouping. Ellen's best treatment was selected as app-based manipulatives. Although she was equally effective with both app and concrete manipulatives with correct digits (100%), she was slightly more independent with the app manipulatives (PND 60%). Ellen also preferred the app manipulatives.

Ellen's accuracy data. During baseline, Ellen's average number of problems correct was 0.4, with a range from 0–2 (refer to Table 1). She answered 100% of the double-digit problems correct in both intervention conditions as well as during best treatment. Compared to baseline data, Ellen's Tau-*U* for accuracy was 100% for both concrete and app-based manipulatives – a large effect. While Ellen's extended baseline showed slight improvement from baseline (μ = 1.8 vs. μ = 0.4), she answered all problems correct during both generalization probes. Ellen's accuracy data for both intervention conditions were stable and zero-celerating.

Ellen's independence data. Like José, Ellen was very independent during all phases of the study, including 100% during baseline, extended baseline (no manipulative condition during the intervention phase), best treatment, and generalization. Across the intervention conditions, Ellen needed few prompts, and never more than a gesture. Specifically, she averaged 98.2% independence across the task analysis steps in the concrete manipulatives condition and 99.4% independence across the task analysis steps in the app-based manipulative condition. Ellen's independence data for both intervention conditions were stable; her data had a zero-celeration trend for the app-based manipulatives and an accelerating trend for the concrete manipulatives.

Ellen's task completion time data. Ellen completed the five subtraction problems during baseline, extended baseline (no manipulative), and generalization phases in less than three minutes. Ellen was faster in solving the five problems with the app-based manipulatives (μ = 8.2 minutes) as compared to the concrete manipulatives (μ = 11.1 minutes). In terms of efficiency, Ellen took, on average, 1.6 minutes per problem with the app-based manipulatives and 2.2 with the concrete manipulatives. Her task completion time during best treatment with the app-based manipulatives was less than during intervention (μ = 7.2 minutes), indicating greater efficiency.

Vince

Like José, Vince struggled with triple-digit subtraction with regrouping. Unlike Ellen and José, though, Vince was more accurate with the concrete manipulatives (PND 40% as compared to a PND of app to concrete of 20%).

Vince indicated he preferred concrete manipulatives.

Vince's accuracy data. Vince averaged 0.4 problems correct during baseline, with a range of 0–1 (refer to Table 1). His average number of problems correct was 4.2 with concrete manipulatives and 4 with app-based manipulatives. Compared to baseline data, Vince's Tau-*U* score was 100% for both concrete and app-based manipulatives – a large effect. Vince answered zero problems correctly during either extended baseline or generalization. He was most accurate during best treatment (μ = 4.7, range 4–5). Vince's data for concrete manipulatives was stable with an accelerating trend; they were decelerating and unstable for the app-based manipulatives.

Vince's independence data. Vince struggled to engage independently in the task analysis steps with both the concrete and app-based manipulatives, although he required fewer steps prompted with the app-based manipulatives (μ = 91.5% vs. 88.0%). He needed multiple levels of prompting, including gesture, indirect verbal, direct verbal, and modeling. Vince's most prompted task analysis steps involved regrouping hundreds or tens blocks. His independence data for both conditions were stable; his app-based manipulative independence data had an accelerating trend while he had a decelerating trend for the concrete manipulatives.

Vince's task completion time data. Vince completed the five subtraction problems during baseline, extended baseline (no manipulative), and generalization phases in less than four minutes. However, he averaged around or over 20 minutes for the two manipulative conditions (μ = 23.4 minutes for concrete and μ = 19.6 minutes for app). In terms of efficiency, Vince took, on average, 4.7 minutes per problem with the concrete manipulatives and 3.9 with the app-based manipulatives. His average task completion time during best treatment with the concrete manipulatives was less than during intervention (μ = 21.7 minutes).

Social Validity

Ellen and José both indicated they preferred the app-based manipulatives, stating they felt they were easier to use despite that they had only used concrete manipulatives previously. Vince, however, preferred the concrete manipulative and indicated he was more comfortable with them. Vince and José expressed that they were both excited for the study to be over because neither enjoyed doing math, while Ellen liked math and liked to work one-on-one with the researchers. The teacher was excited about the potential of the app-based base 10 blocks and was going to look into seeing if it might be able to be purchased and additional iPads secured for her students. She also stated that she wished at least one other student returned a consent form as she felt he too could have benefited from additional one-on-one work with subtraction.

Discussion

This study explored how app-based manipulatives compared to concrete manipulatives in supporting students with disabilities in solving subtraction problems with regrouping. To compare both manipulatives in terms of accuracy, independence, and task completion time, researchers used a single subject adapted alternating treatment design with three middle school students – two with mild intellectual disability and one with a learning disability. The main result is that app-based base 10 blocks can be just as effective and efficient as concrete base 10 blocks.

Although the results were idiosyncratic in terms of what type of manipulative was more effective, all three students improved in terms of correctly solving the problems during the intervention conditions as compared to baseline. These results are consistent with previous research that suggests manipulatives – both concrete and app-based – are effective for students with disabilities (c.f., Bouck et al., 2014; Satsangi & Bouck, 2015). In addition, both types of manipulatives were able to support students; there was not a clear, consistent difference in terms of accuracy between the two types of manipulatives. In other words, it could be suggested that app-based manipulatives could equally support students with disabilities as compared to concrete manipulatives, also consistent with previous research (Bouck et al., 2014; Satsangi et al., 2016).

Despite prompting and use of manipulatives, Vince and José did not always achieve

100% accuracy during the intervention conditions. While the researchers prompted for students to set up the problems correctly in terms of following the task analysis steps, if students made a subtraction error, they were not prompted to correct; Vince and José made subtraction errors. With the concrete manipulatives, errors involved removing too many ones blocks during the subtraction, and with the app-based manipulatives, particularly with Vince, the errors involved inherent elements within the app. Two issues were at play. One was when subtracting with the app-based manipulatives, the blocks would become semi-transparent (or ghost out), and Vince would struggle to see all the opaque blocks. The other issue involved the small screen size; when there were 15 tens blocks, for example, they could be difficult to see or even hidden and sometimes they would be missed in the counting after the subtraction occurred.

In terms of independence, students needed more prompts during intervention as compared to baseline and extended baseline (i.e., no manipulatives). However, even though students answered some problems correctly during baseline and extended baseline, they were not consistently accurate. Overall, Ellen and José needed few prompts during the intervention conditions and zero during their best treatment of app-based manipulatives. When prompts were given, it was almost always a gesture and related to regrouping. Regrouping during subtraction was a challenge for all three students, and, when analyzing their digits correct across all phases, the reason they were largely incorrect without the manipulatives. Regrouping is a challenging concept for students, including many students with disabilities (Witzel, Ferguson, & Mink, 2012). There is evidence from the accuracy scores during generalization and, to some extent, extended baseline that suggest Ellen and José were beginning to internalize the regrouping step in their subtraction problems even when manipulatives were not present. Vince, on the other hand, needed more prompting. His performance was consistent with his processing challenges. The researchers also found that the 10-second delay reduced the numbers of prompts that might otherwise be given, as he often performed a step at 8 or 9 seconds within the time delay.

Students were more independent in solving the problems during baseline; they all experienced an increased amount of prompting – or conversely a decrease in independence – during intervention as compared to baseline. Although minimal for Ellen and José, one hypothesis is that students may have needed more training with both types of manipulatives prior to entering the intervention phase. However, Ellen and José both experienced the slight increase in prompting during the middle of the intervention period for both manipulatives, rather than the beginning. These results suggest that perhaps if the intervention sessions occurred more frequently (i.e., more than twice a week or without breaks dues to absences), Ellen and José would have been more independent. The more training hypothesis could be appropriate for Vince, given that he need more prompting for the app-based manipulative during the first session.

All three students were more independent with the app-based manipulatives. One hypothesis for these data is that the virtual – or app-based – manipulatives have built in constraints or structure to further support the student. For example, with the app-based manipulatives, one cannot regroup a tens block from the subtrahend as it is prohibited within the app. With the concrete manipulatives, Vince more than once attempted to regroup from the subtrahend rather than the minuend and the researchers would need to prompt. Hence, any form of virtual manipulatives may work to reduce students' cognitive load (Suh & Moyer, 2008).

Based on the accuracy, independence, and task completion time data, the researchers hypothesize that Ellen and José could be supported in learning subtraction with regrouping through the Concrete-Representational-Abstract (CRA) approach; the CRA approach is considered an evidence-based or effective approach for teaching mathematical concepts to students with disabilities (Agrawal & Morin, 2016). The researchers further hypothesize both Ellen and José could be taught the CRA approach with app-based manipulatives substituted in place of concrete. Not only was this deemed to be the best treatment condition for both students, but it was also their preferred condition. Ellen and José both indicated a preference for app-based manipulatives, given their perception that they were easier and, as supported in other studies, more age appropriate and less socially stigmatizing (Satsangi & Bouck, 2015).

Implications for Practice

This study suggests implications for practice. For one, it suggests students with disabilities – such as mild intellectual disability and learning disability – can be supported in solving mathematical problems with app-based manipulatives. The use of an alternative to concrete manipulatives – while beneficial – is particularly important for older children, given that many manipulatives may not be age appropriate for secondary students and undesirable socially (e.g., concrete base 10 blocks). App-based manipulatives allow older students to be supported in a less conspicuous manner, and hence, students may be more likely to use the tool. Second, it does suggest that app-based and concrete manipulatives may be inter-changeable and thus teachers implementing the CRA approach for students with disabilities or struggling students may be able to use app-based manipulatives in place of concrete within the sequence.

Limitations and Future Directions

This study has multiple limitations. With the adapted alternating treatment design was used, solving the math problems with concrete and app-based manipulatives were not probed during baseline. The researchers only probed for no manipulatives during baseline and extended it throughout the intervention as the control set (Wolery et al., 2010). One reason for not probing the manipulative conditions during baseline was the repeated exposure students had previously to the concrete manipulatives. However, the differential amount of exposure (i.e., they had never used app-based manipulatives prior to the study while the teacher routinely used concrete manipulatives) could be considered another limitation. In addition, a pre-determined set of criteria was not used to determine the intervention phase (Wolery et al., 2010). Another limitation involves that the researchers implemented the system of least prompts along with the intervention conditions; the impact of the system of least prompts was not separated. A final limitation involves that Vince, in particular, may have benefited from additional training prior to entering the intervention, especially with the app-based manipulatives.

In terms of future research, researchers should continue to explore the use of app-based manipulatives to support students with disabilities in mathematics, including conducting group design research. Researchers should also extend the exploration of app-based manipulatives to other mathematical areas, such as area and perimeter and algebra. In addition, given the few minor concerns regarding the actual app selected, researchers should explore other app-based base 10 manipulatives as their effectiveness and efficiency in supporting students with disabilities in subtraction, or even addition or multiplication. Finally, researchers should examine the use of virtual – including app-based and online – manipulatives within the approach, given the effectiveness of CRA in supporting students with disabilities.

References

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with disabilities. *Learning Disabilities Research & Practice, 31,* 34–44. doi:10.111/ldrp.12093

Bouck, E. C., & Flanagan, S. M. (2010). Virtual manipulatives: What are they and how teachers can use them? *Intervention in School and Clinic, 45,* 186–191.

Bouck, E. C., Flanagan, S. M., & Bouck, M. K. (2015). Learning area and perimeter with virtual manipulatives. *Journal of Computers in Mathematics and Science Teaching, 34,* 381–393.

Bouck, E. C., Satsangi, R., Doughty, T. T., & Courtney, W. T. (2014). Virtual and concrete manipulatives: A comparison of approaches for solving mathematics problems for students with autism spectrum disorder. *Journal of Autism and Developmental Disabilities, 44,* 180–193. doi:10.1007/s10803-013-1863-2

Brainingcamp, LLC. (2016). Base Ten Blocks Manipulative (Version 1.1.0) [Mobile application software].

Bryant, B. R., Ok, M., Kang, E. Y., Kim, M. K., Lang, R., Bryant, D. P., & Pfannestiel, K. (2015). Performance of fourth-grade students with learning disabilities on multiplication facts comparing teacher-mediate and technology-mediate intervention: A preliminary investigation. *Journal of Behavioral Education, 24,* 255–272. doi:10.1007/s10864-015-9218-z

Doabler, C. T., & Fien, H. (2013). Explicit mathematics instruction: What teachers can do for teaching students with mathematics difficulties. *Intervention in School and Clinic, 48,* 276–285. doi: 10.1177/1053451212473141

Doyle, P. M., Wolery, M., Ault, M. J., & Gast, D. L. (1988). System of least prompts: A literature review of procedural parameters. *Journal of the Association for Persons with Severe Handicaps, 13*(1), 28–40.

Gast, D. L., & Spriggs, A. D. (2010). Visual analysis of graphic data. In D. L. Gast (Ed.), *Single subject research methodology in behavioral sciences* (pp. 199–233). New York: Routledge.

Lai, S. A., & Berkeley, S. (2012). High-stakes test accommodations: Research and practice. *Learning Disability Quarterly, 35,* 158–169. doi:10.1177/0731948711433874

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*(5), 1–22.

Marley, S. C., & Carbonneau, K. J. (2014). Future directions for theory and research with instructional manipulatives: Commentary on the special issue papers. *Educational Psychology Review, 26,* 91–100. doi: 10.1007/s10648-014-9259-1

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of mathematical representations. In W. J. Masalski & P. C. Elliott (Eds.), *Technology-supported mathematics learning environments: 67th yearbook* (pp. 17–34). Reston, VA: NCTM.

Ok, M. W., & Bryant, D. P. (2015). Effect of strategic intervention with iPad practice on multiplication fact performance of fifth-grade students with learning disabilities. *Learning Disability Quarterly* [Advanced Online Publication]. doi:10.1177/0731948715598285

Parker, R. I., Vannest, K. J., & Brown, L. (2009). An improved effect size for single case research: Nonoverlap of all pairs (NAP). *Behavior Therapy, 40,* 357–367.

Parker, R. I., Vannest, K. J., Davis, J. L., & Sauber, S. B. (2011). Combining non-overlap and trend for single case research: Tau-U. *Behavior Therapy, 42,* 284–299.

Pilgrim, J., Bledsoe, C., & Riley, S. (2012). New technologies in the classroom. *The Delta Kappa Gamma Bulletin, 78*(4), 16–22.

Satsangi, R. (2015). *Comparing the efficacy of virtual and concrete manipulatives to learn algebra for secondary students with learning disabilities* (Unpublished doctoral dissertation). Purdue University, West Lafayette, IN.

Satsangi, R., & Bouck, E. C. (2015). Using virtual manipulative instruction to teach the concepts of area and perimeter to secondary students with learning disabilities. *Learning Disability Quarterly, 38,* 175–186. doi:10.1177/0731948714550101

Satsangi, R., Bouck, E. C., Taber-Doughty, T., Bofferding, L., & Roberts, C. A. (2016). Comparing the effectiveness of virtual and concrete manipulatives to learn algebra for secondary students with learning disabilities. *Learning Disability Quarterly.* [Advanced Online Publication].

Sindelar, P. T., Rosenberg, M. S., & Wilson, R. J. (1985). An adapted alternating treatments design for instructional research. *Education and Treatment of Children, 8*(1), 67–76.

Suh, J. M., & Moyer, P. S. (2008). Scaffolding special needs students' learning of fraction equivalence using virtual manipulatives. *Proceedings of the International Group for the Psychology of Mathematics Education, 4,* 297–304.

The Nation's Report Card. (2016). *2015 mathematics & reading assessment.* Retrieved from, http://www.nationsreportcard.gov/reading_math_2015/#?grade=4

Vannest, K. J., Parker, R. I., & Gonen, O. (2011). *Single case research: Web-based calculators for scr analysis.* (Version 1.0) [Web-based application], College Station, TX: Texas A&M University. Retrieved from, http://www.singlecaseresearch.org/calculators

White, O. R., & Haring, N. G. (1980). *Exceptional teaching* (2nd ed.). Columbus, OH: Merrill.

Witzel, B. S., Ferguson, C. J., & Mink, D. V. (2012). Number sense: Strategies for helping preschool through grade 3 children develop math skills. *Young Children, 67,* 89–94.

Wolery, M., Gast, D. L., & Hammond, D. (2010). Comparative intervention designs. In D. L. Gast (Ed.), *Single subject research methodology in behavioral sciences* (pp. 329–381). New York: Routledge.

Received: 18 May 2016
Initial Acceptance: 5 July 2016
Final Acceptance: 25 August 2016