Learning Disabilities Research & Practice, *29*(4), 171–183

Teaching Multiplication with Regrouping to Students with Learning Disabilities

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The Common Core Standards require demonstration of conceptual knowledge of numbers, operations, and relations between mathematical concepts. Supplemental instruction should explicitly guide students with specific learning disabilities (SLD) in these skills. In this article, we illustrate implementation of the concrete-representational-abstract (CRA) sequence and the Strategic Instruction Model (SIM) for teaching multiplication with regrouping to students with SLD. CRA combined with SIM has been shown to be effective in teaching computation for students with SLD, specifically for developing conceptual understanding. Four elementary students with SLD participated in this study. The researchers used a multiple-probe design to show a functional relation. Students demonstrated increases in computational fluency; skills were maintained and generalized.

The importance of mathematics achievement is indisputable and highlighted by multiple reform efforts across the country over the past two decades. These reform efforts have focused on improving mathematics achievement in the United States throughout the school-age years. One result of these reform efforts is the development of the Common Core State Standards (CCSS). The CCSS have currently been adopted by 45 states, the District of Columbia, four territories, and the Department of Defense Education System (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

The CCSS outline grade-specific expectations for students and serve as a guide for teachers to follow when planning their instruction in core academic areas. The focus of the CCSS in mathematics standards is conceptual understanding, although procedural knowledge and fluency are also seen as important (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Porter, McMaken, Hwang, & Yang, 2011). For example, fluency in basic operations and procedural knowledge are necessary in the development of mathematical thinking and conceptual understanding of the basis of numbers and operations in order to explain why particular operations or procedures are used to solve a problem. The CCSS focus on a deep understanding of how to complete operations, understanding relationships between operations and articulation of how operations are completed. Because a majority of students with mathematics difficulties are expected to master the mathematical practices included in the CCSS, it is important to recognize common difficulties students with specific learning disabilities (SLD) in mathematics experience in order to differentiate instruction.

Students who have SLD in mathematics or who have poor mathematics achievement may struggle to meet the conceptual focus of the CCSS due to their learning characteristics. Difficulties experienced by students with SLD include organization of information, understanding and using learning strategies, acquiring basic computational skills necessary for higher order mathematics operations, connecting new material to previously learned material, solving word problems, and communicating about mathematic processes (Doabler et al., 2012; Garnett, 1987; Garnett, 1998; Geary, 2004; Hudson & Miller, 2006; Jitendra, 2013; Mancl, Miller, & Kennedy, 2012).

Although the CCSS provide instructional objectives for teachers to use for planning, they do not include directions or activities to facilitate instruction in the standards. Therefore, it is up to teachers to design and implement lessons that meet the needs of all students, including students with poor mathematics performance and/or SLD. Current research on effective mathematics instruction identifies several effective strategies for students who have mathematics difficulties (Hudson & Miller, 2006; Miller, Stringfellow, Kaffar, Ferreira, & Mancl, 2011; Peterson, Mercer, & O'Shea, 1998). One strategy is the Concrete-Representational-Abstract (CRA) approach. This approach provides explicit instruction and emphasizes

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conceptual understanding as recommended by the National Mathematics Advisory Panel (2008). The CRA approach is different than other general education conceptual models of instruction such as the use of area models (Van de Walle, Karp, Lovin, & Bay-Williams, 2010) which use manipulatives and show the operation by filling an area with base-ten manipulative blocks. CRA fills the gap between conceptual understanding of the overall operation (accomplished with area models) and the shortened widely accepted algorithms used to solve computation problems. CRA bridges this gap by teaching algorithmic procedures using manipulatives and drawings, and by eventually fading these aids until students solve problems fluently using numbers only. CRA provides students with tools to show and describe how operations are completed within a base-ten number system which is consistent with the CCSS related to conceptual understanding.

The theoretical underpinnings of CRA are the Stages of Representation (Bruner & Kenney, 1965). These sequenced stages include learning through action and manipulation of objects (enactive), learning through pictures (iconic), and learning through symbols (symbolic). Accordingly, CRA involves the following three instructional phases. First, the concrete phase involves explicit instruction using manipulatives to represent numbers. The teacher models the skill with the use of manipulatives and provides opportunities for guided practice and independent practice. Next, during the representational phase, the teacher uses the same instructional sequence using pictures to represent numbers instead of the manipulatives. Finally, during the abstract phase, the teacher and students complete problems using numbers only and the emphasis is automaticity and fluency using numbers only.

CRA has been combined with the Strategic Instruction Model (SIM), a method that includes explicit instruction and emphasizes procedural knowledge. The addition of SIM provides further scaffolding in moving from iconic understanding to symbolic understanding (Bruner & Kenney, 1965). The combination of CRA-SIM addresses learning deficits commonly demonstrated by students with disabilities, including difficulties with organization of information and poor access or utilization of long- and short-term memory (Deshler & Hock, 2006).

Several studies have shown that CRA combined with SIM (CRA-SIM) is an effective practice when teaching basic computation. Mercer and Miller (1992), found the *Strategic Math Series* curriculum that uses CRA procedures and SIM using a mnemonic strategy (Discover the sign, Read the problem, Answer or draw and check, Write the answer; DRAW) to be more effective than the traditional curriculum when teaching students with disabilities to acquire, understand, and apply basic addition, subtraction, multiplication, and division. Miller and Mercer (1993) replicated these findings with elementary students with SLD. Other research has shown CRA-SIM to be effective in increasing performance with regard to basic operations (Harris, Miller, & Mercer, 1995; Morin & Miller, 1998), integers, fractions, and algebra (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003).

In more recent research, CRA-SIM was shown to be effective in teaching complex addition and subtraction with regrouping to students with disabilities as well as to students at risk for mathematics failure. Flores (2009, 2010) taught subtraction with regrouping to students experiencing mathematics difficulty and students at risk for mathematics failure using CRA-SIM. She used a multiple probe across participants design. Instruction included CRA methods using explicit instruction and the DRAW mnemonic strategy. Her results showed a functional relation between CRA-SIM and positive learning outcomes for subtraction with regrouping across all students.

Miller & Kaffar (2011) taught addition with regrouping to students with learning difficulties in mathematics. They compared instruction using CRA-SIM instruction with a traditional basal curriculum. The treatment group received 16 lessons of CRA-SIM including explicit instruction using the CRA sequence and procedural strategy (Read the problem, Examine the ones' column, Note ones in the ones' column, Address the tens' column, Mark tens in the tens' column, Examine and note hundreds and exit with a quick check; RENAME). During the concrete and abstract levels, the students solved problems using a place value mat that served to assist in organizing manipulatives (base-ten blocks) and pictures. Two lessons involved teaching mnemonic strategies during the transition to the abstract phase. The comparison group received 16 lessons from a traditional second grade basal series. Students in the treatment group performed significantly better in addition with regrouping than the comparison group. This study showed that CRA-SIM was more effective than a basal curriculum.

Mancl et al. (2012) also used a multiple probe across participants design to study the use of CRA-SIM to teach subtraction with regrouping to students with SLD. The researchers used CRA methods and the taught the RENAME strategy during the transition phase. Through the CRA sequence of instruction, the students solved problems using a place value mat, base-ten blocks, and pictures. Then, students progressed to problem solving using numbers only. Results indicated a functional relation between CRA-SIM and increased accuracy in subtraction. Although CRA-SIM has been extended to addition and subtraction, there is a lack of research in the area of multiplication with regrouping. Multiplication with regrouping involves application of conceptual knowledge of numbers and operations in order for a student to explain how or why particular procedures are used in solving such problems. Students with SLD need interventions that support the mathematical practices within CCSS, providing conceptual understanding of operations while developing procedural knowledge and fluency. Therefore, the purpose of this study was to use CRA-SIM to teach multiplication with regrouping to students with SLD. The research question was: What are the effects of CRA-SIM on the multiplication performance of students with SLD when solving problems that include two-digit multipliers?

METHOD

Setting

The study took place in a special education resource classroom at a rural elementary school in the Southeastern United States. The researcher taught each student individually during his/her mathematics class scheduled with the special education teacher outside of the general education classroom. Instructional sessions were 25 minutes in duration 3 days per week during which time the researcher administered a timed probe and implemented instruction.

Participants

The participants were four elementary students who qualified for special education services under SLD. The State eligibility criterion includes a regression-based discrepancy model, failure within a response-to-intervention framework, or a pattern of strengths of weaknesses. Each student received services for reading and mathematics. Mari and Jon were in the fourth grade; Ed and Jack were in fifth grade. They were chosen based on the following: (1) eligible for special education services in the area of SLD; (2) fluent in basic addition, subtraction, and multiplication as defined as writing at least 45 correct digits per minute; (3) mastery of multiplication with regrouping with problems that included a one-digit multiplier as defined as writing 30 correct digits in 2 minutes; and (4) inability to compute multiplication with regrouping problems that included a two-digit multiplier as defined as less than 10 percent of problems correct. All students met this criterion. The students partially completed two-digit multiplication problems by attending to the digit in the ones' place of the multiplier; error patterns varied. Consistent with the recommendations for participant descriptions by Rosenberg et al. (1993), the students' eligibility with model used, computation achievement, cognitive functioning, and background are located in Table 1.

Materials

The assessment materials were probes that included 25 problems requiring multiplication with regrouping of 2 two-digit numbers. There were four probes, sheets of paper with the problems printed in 12-point font. Prior to the study, the researchers developed four versions of multiplication probes used for assessment during baseline, intervention, and maintenance. The researchers used four versions in order to control for practice effects; the students received different versions in different orders across sessions. During development, the researchers collected data regarding reliability and content validity. The problems within the probes were administered to college students to ensure that problems would be completed consistently. There was a Cronbach's Alpha Coefficient of $r = .73$ for probe items. For content validity, the researchers computed the item-level content validity index (I-CVI) through a process in which experts (three elementary mathematics teachers with at least 4 years of experience and a Master's degree) rated problems' relevance to the skill. The researchers divided these ratings by the total number of experts (Lynn, 1976) yielding an I-CVI of 1.00, meaning that the problems were highly relevant to the skill. In addition, two teachers and one state department mathematics instructional specialist rated the same problems as *easy, average,* or *difficult* for the typical elementary school student. The items were rated as *average*. The I-CVI calculation provided rationale for the probes' content. The ICVI showed that problems were similar to those used for general assessment of multiplication with regrouping with regard to relevance and ease of calculation.

Instructional materials for the CRA_SIM consisted of the following: (1) an instructional manual created by the first author that provided instructional procedures and suggested scripts for each lesson, and an outline of teacher behaviors for the provision of the advance organizer, guided practice, independent practice, and provision of a post-organizer for each lesson determined prior to the study; (2) student learning sheets for each lesson; (3) place value mats used during concrete and representational instruction; and (4) base-ten blocks used during concrete instruction. The student learning sheets included three sections with problems used for teacher demonstration, guided practice, and independent practice. The learning sheet for lesson seven differed from the others; it had the RENAME strategy printed in the middle of the page. The place value mats were laminated pages used to organize base-ten blocks or drawings when solving problems at the concrete and representational levels; these mats were not used during abstract level instruction. The place value mat for concrete level instruction was a 36×36 inch sheet which provided the student with space to organize base-ten blocks. The place value mat for representational level instruction was a 16×16 inch sheet which provided the student with space to organize drawings while solving problems. The mats differed in size because base-ten blocks required a larger amount of space than drawings. The larger mat required much table space, so when base-ten blocks were no longer required, the students used the smaller mat because it took less tabletop space. Both types of place value mats were tables that had columns labeled for the ones', tens', hundreds', and thousands' places. The columns were divided into three rows, each shaded to visually differentiate multiplication by the multiplier digit in the ones' place, tens' place, and total when both rows were added. Within each column row, the cells were further divided into nine smaller cells for grouping because one could form up to nine groups (of one, ten, or hundred) without regrouping. A visual example of the description above is provided in step 9 of Figure 1.

Procedures

Assessment

Prior to instruction, to establish baseline, students completed timed (2-minute) probes. After instruction began, probes were administered at the beginning of lessons prior to any instruction until students reached the criterion for mastery. The number of probes varied by student based on the number of sessions given prior to mastery; the researchers gave 10 probes to Mari and Jon, 7 probes to Ed, and 12 probes to Jack. After instruction ended, probes were administered to assess maintenance. The teacher placed the probe in front of the student and told him/her to complete as many problems as he/she could until told to stop. The teacher told the student to begin, started a timer, and asked the student to stop

TABLE 1
Student Demographics

Student	*Age*	*Grade*	*Cultural Background*	*Eligibility (model)*	*Hours Served per Week*	*Cognitive Ability (IQ)[a]*	*Mathematics Computation Achievement[b]*
Mari	10	4	African American	Specific Learning Disability (response to intervention)	10	90	86
Jon	10	4	Latino	Specific Learning Disability (discrepancy)	10	105	86
Ed	11	5	White	Specific Learning Disability (discrepancy)	10	103	80
Jack	11	5	White	Specific Learning Disability (discrepancy)	10	101	72

a = standard score reported in most recent special education evaluation or re-evaluation.
b = standard score Operations subtest *Key Math 3 Diagnostic Assessment* (Connolly, 2007)

after 2 minutes. Baseline procedures involved the administration of probes only. Instruction began when the first student demonstrated a stable baseline, as defined as at least five data points with the last three data points varying no more than 20 percent from the mean of the baseline data path. The other students remained in baseline until the first student wrote at least 25 correct digits on a probe. The researchers chose the criterion of 25 digits correct for phase change (rather than 30 digits) because that amount would show a clear increase over baseline and allow the study to be completed without interruption via annual testing, field trips, festivals, and so forth during the school year. Digits correct were defined as the number of digits written below the horizontal line beneath the original problem (the answer, thus), as well as all of the digits used to calculate the problem. This metric showed small increments in learning progress across all of the procedural steps (Hosp, Hosp, & Howell, 2007). For example, if the student completed portions of the algorithm correctly, but failed to arrive at the correct answer, this metric showed an increase in digits correct. Although percentages of correct answers may be more common within general education assessment, the researchers chose digits correct because it is the most sensitive measure of student progress toward fluency (Keller-Margulis, Shapiro, & Hintze, 2008).

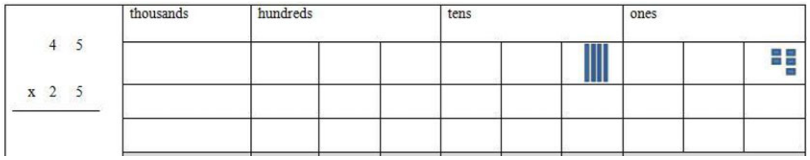
Concrete Lesson Procedures

Across all phases of instruction, lessons were implemented according to the explicit instruction model with an advance organizer, teacher demonstration, guided practice, independent practice, and a post-organizer. The first three lessons involved instruction at the concrete level using base-ten blocks. The concrete level is shown through pictures of one example problem in Figure 1.

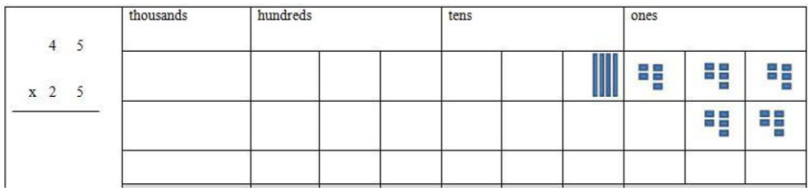
During each concrete level instructional lesson, the teacher modeled two problems, only asking students to provide information that they already knew or repeat information stated by the teacher. The teacher conducted guided practice with two problems, working together with students to solve problems in a back and forth process. The students solved two problems for independent practice. The teacher did not assist in independent problem solving, but checked the students' work and provided verbal feedback when students finished the problems. The amount of time and effort required for solving problems at the concrete level determined the number of problems within lessons. The problem solving steps are summarized in Table 2 and a more detailed description follows.

The teacher reviewed the reverse rule or commutative property; employing this property allowed for more efficient computation and manipulation of objects (e.g., four groups of 20 rather than 20 groups of four). The teacher read the problem aloud and represented the multiplicand (top number) on the place value mat as shown in the first portion or step of Figure 1. Beginning in the ones' column, the teacher represented problem, making the appropriate number of groups (second step of Figure 1.). The teacher evaluated the answer and if there were ten or more ones, regrouping occurred by exchanging 10 ones for 1 tens' block which was placed in the tens' column (third step of Figure 1). The teacher also noted regrouping on the written problem. Next, the tens' place of the multiplicand was multiplied by the ones' place of the multiplier and the teacher represented this problem on the place value mat (fourth step of Figure 1). The teacher evaluated the answer, and if there were ten or more tens, regrouping occurred by exchanging 10 tens for 1 hundreds' block which was placed in the hundreds' column (fifth step of Figure 1). The teacher noted numbers in the written problem. Before multiplying by the tens' place of the multiplier, the teacher/student crossed out the number in the ones' place of the multiplier and wrote a zero in the ones' place underneath the first line of answers within the problem. Next, the teacher multiplied the number in the ones' place of the multiplicand by the number in the tens' place of the multiplier. The teacher employed the commutative property for ease of problem solving. The teacher represented the problem using base-ten blocks (sixth step of Figure 1). The teacher evaluated the answer for regrouping, and if there were ten or more, 10 tens were exchanged for a hundreds' block which was

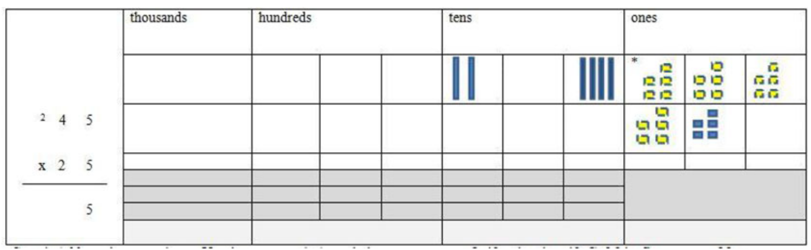
Step 1. Represent the multiplicand (top number) with base ten blocks.



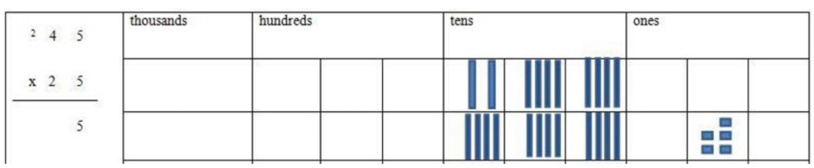
Step 2. Examine the ones' place. Multiply one's place of multiplicand (top) by multiplier (bottom); five groups of five.



Step 3. Note the ones. If there are ten or more, go next door. Remove two groups of ten ones and add two tens to the to the tens' place. Note five ones. * Pictures with dashed lines represent objects that were used in regrouping, removed and exchanged for larger number.



Step 4. Address the tens' column. Use the reverse rule (associated property, e.g., 5x40 rather than40x5), Make five groups of four tens.



Step 5. In noting the tens, if there are ten or more, go next door. Remove two groups of 10 tens and add two hundreds to the hundreds' place. Mark the 2 tens. Examine and note the two hundreds. Begin again in the tens' place of the multiplier.

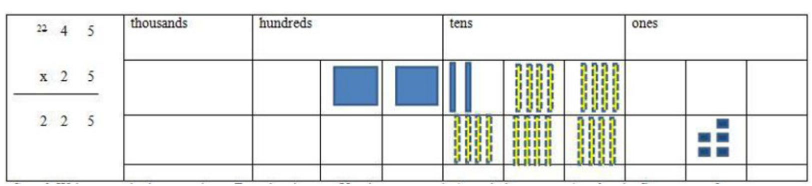
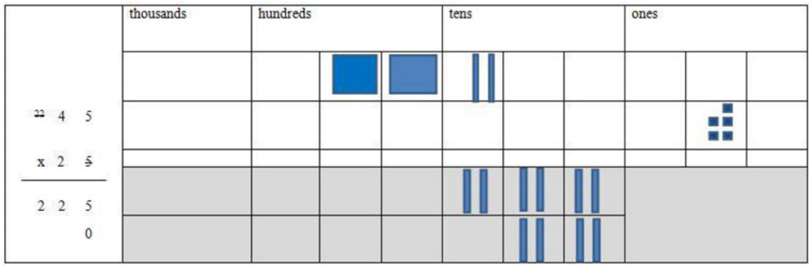


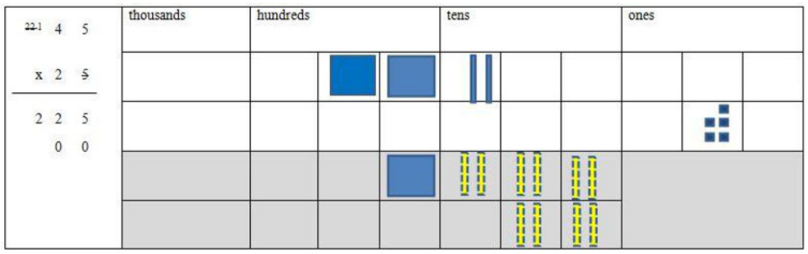
FIGURE 1 Procedures for solving two-digit by two-digit multiplication problems at the concrete level.

placed in the hundreds' place on the mat. The teacher noted regrouping on the written problem (seventh step of Figure 1). The numbers in the tens' places of the multiplicand and the multiplier were multiplied (e.g., 40 groups of 20). This was an unwieldy problem using manipulatives, so the teacher solved it using tens' blocks one time for demonstration and then taught the following short cut. The teacher demonstrated that 40 groups of 20 was the same as four groups of two hundreds (eighth step of Figure 1). The teacher demonstrated this concept, guided the students in demonstrating the concept, and the students demonstrated this independently. The students were satisfied that using hundreds' blocks to solve this problem was easier than using many groups of tens' blocks. Once represented and solved, the teacher evaluated the

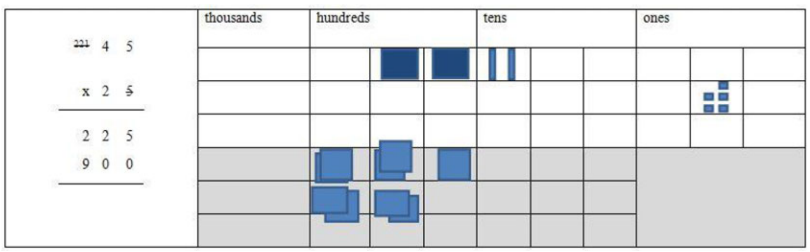
,Step 6. Write a zero in the ones' place. Examine the tens. Use the reverse rule (associated property) and make five groups of 2 tens.



Step 7. Regrouping is required. Remove 10 tens and add one hundred. Mark the tens' place.



Step 8. Address the hundreds'. Making 40 groups of 20 is the same as four groups of two hundreds. Mark the hundreds' place (nine).



Step 9. Add the numbers and examine the problem to ensure that blocks match the numbers.

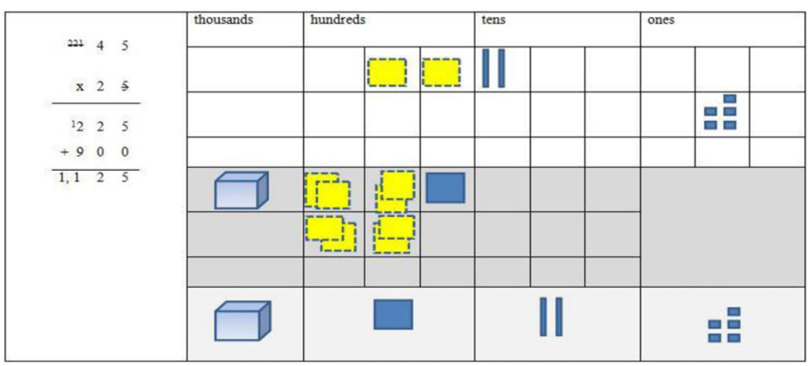


FIGURE 1 Continued.

answer for regrouping. If there were ten or more hundreds, they were exchanged for a thousands' block. The teacher wrote or noted the numbers. Finally, the teacher added the answers obtained by multiplying by both numbers in the multiplier. She used the place value mat and written problem; when adding each column, regrouping was necessary at times as seen in the hundreds column within Figure 1. Finally, the teacher compared the manipulatives on the mat with the answer in the written problem (ninth step of Figure 1).

Representational Lesson Procedures

The procedures for instruction at the representational level (lessons four through six) involved drawings rather than base-ten blocks. Ones were small tallies drawn on a horizontal line. Tens were long vertical lines. Hundreds were squares, and thousands were cubes. Instructional procedures regarding modeling, guided practice, and independent practice were the same for representational level instruction as for concrete

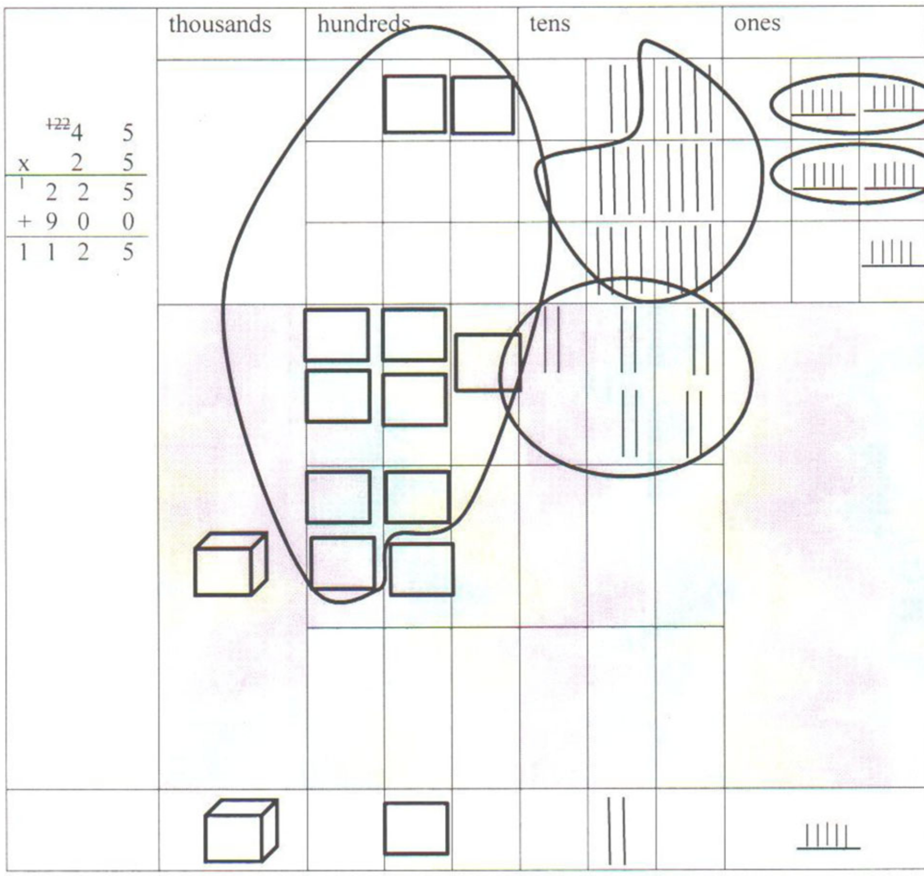


FIGURE 2 Completed problem at the representational level.

level instruction. The teacher and students represented regrouping by circling portions of drawings when exchanged (e.g., 10 ones circled and 1 vertical line added to tens' column). The teacher modeled three problems, guided three problems, and the students completed three problems independently. A completed problem at representational level is shown in Figure 2.

Abstract Lesson Procedures

The seventh lesson involved teaching a strategy for solving regrouping problems. The strategy was: (1) Read the problem; (2) Examine the ones; (3) Note the ones; (4) Address the tens; (5) Mark the tens; and (6) Examine the hundreds and note the hundreds; exit the first line and begin again or add and check (RENAME). Instruction with the RENAME strategy involved verbal rehearsal until the student could look at the first letter of the mnemonic and state the strategy step. Abstract level instruction (lessons eight through ten) involved the use of numbers only and the RENAME strategy. Using the explicit instruction steps, the teacher and student solved problems with the RENAME mnemonic within sight, but no other aids. During lessons eight through ten, the teacher modeled two problems, guided three problems, and the students solved four problems independently.

Maintenance and Generalization

Two weeks after the students mastered regrouping and instruction ended, probes were administered to measure maintenance. Additional maintenance probes were given each week following for two additional weeks, but assessment across students differed since instruction was staggered. Jon completed maintenance probes 2 weeks, 3 weeks, and 4 weeks after instruction. Ed completed maintenance probes 2 weeks and 3 weeks following. The researchers administered a generalization probe at the end of the study. The generalization probe consisted of regrouping problems with a three-digit multiplicand and the two-digit multiplier; instruction did not include these problems, but involved similar procedures.

Treatment Fidelity, Inter-observer Agreement, and Social Validity

Fidelity measures met or exceeded those recommended by Horner Carr, Halle, McGee, Odom, and Wolery (2005). The second and third authors collected treatment fidelity data through live observations using a checklist of teacher behaviors associated with probe administration and CRA-SIM instruction. The observer completed checklists two out of

TABLE 2
Summary of Concrete Level CRA-SIM Instructional Steps

• Review the commutative property
• Read the problem aloud and represent the multiplicand (top number) on the place value mat.
• Beginning in the ones' column, represent problem, making the appropriate number of groups.
• Evaluate answer. If there are 10 or more ones, regroup by exchanging 10 ones for 1 tens' block and place in the tens' column. Note regrouping on the written problem.
• Multiply the tens' place of the multiplicand by the ones' place of the multiplier, represent this problem on the place value mat.
• Evaluate the answer, and if there are ten or more tens, regroup by exchanging 10 tens for 1 hundreds' block, place in the hundreds' column. Note numbers in the written problem.
• Cross out the number in the ones' place of the multiplier and write a zero in the ones' place underneath the first line of answers within the problem.
• Multiply the number in the ones' place of the multiplicand by the number in the tens' place of the multiplier (employ the commutative property for ease of problem solving).
• Represent problem and evaluate for regrouping. If there were ten or more, exchange 10 tens for a hundreds' block and place in the hundreds' place on the mat. Note on written problem.
• Multiply the numbers in the tens' places of the multiplicand and the multiplier (e.g., 40 groups of 20). Instead, represent this problem as four groups of two hundreds.
• Evaluate the answer for regrouping. If there were ten or more hundreds, exchange for a thousands' block. Note the numbers on the written problem.
• Using place value mat and written problem, add the answers obtained by multiplying by both numbers in the multiplier. Compare the manipulatives on mat with the written answer.

three (66 percent) sessions per week; therefore data were collected across all students, during baseline, and across all instructional phases (concrete, representational, and abstract levels). The researchers calculated treatment fidelity of 100 percent throughout the study.

The researchers checked probes for consistency through inter-scorer accuracy. A second observer scored 50 percent of all probes during baseline and instructional phases. The researchers calculated inter-scorer agreement by adding the number of agreements and dividing that sum by the total number of agreements and disagreements. Agreement was 100 percent for Mari's probes, 100 percent agreement for Jon's probes, 97 percent agreement for Jack's probes, and 100 percent agreement for Ed's probes.

The researchers assessed social validity using open-ended written questionnaires distributed to the resource teacher and read to the students before and after the study. The items on the questionnaires asked about the students' multiplication performance, need for intervention, thoughts about CRA-SIM, students' performance after the study, and recommendations for others. On the questionnaire given prior to the study, the students and their teacher reported the following: (1) multiplication with regrouping was difficult, (2) students performed poorly in that area, and (3) there was a need for additional instruction. After the study, the students and teacher reported: (1) multiplication with regrouping was easy and fun, (2) performance improved, and (3) they would participate again and recommend CRA-SIM to others.

Research Design

The researchers used a multiple probe across students design to investigate the presence of a functional relation between CRA-SIM instruction and students' multiplication with regrouping performance. All students began baseline and the first student, Mari, moved from baseline to instruction after demonstrating stable performance. Jon began instruction after Mari reached the criterion for phase change (25 correct digits) and his baseline performance was stable. Ed and Jack were enrolled in the same class and, because of scheduling, moved from baseline to intervention together after Jon reached the criterion for phase change; they demonstrated stable baselines. In order to evaluate the presence of a functional relation, researchers inspected student data with attention to the level, trend, and overlap of data paths.

RESULTS

Baseline

Baseline data were collected across all students and data paths remained stable for all students prior to instruction. The data points on each graph represent student performance on probes administered prior to instructional lessons across all phases of CRA-SIM. The first data point for each student represents his/her performance on the probe given prior to instruction for the second lesson. The results for all students are located in Figure 3.

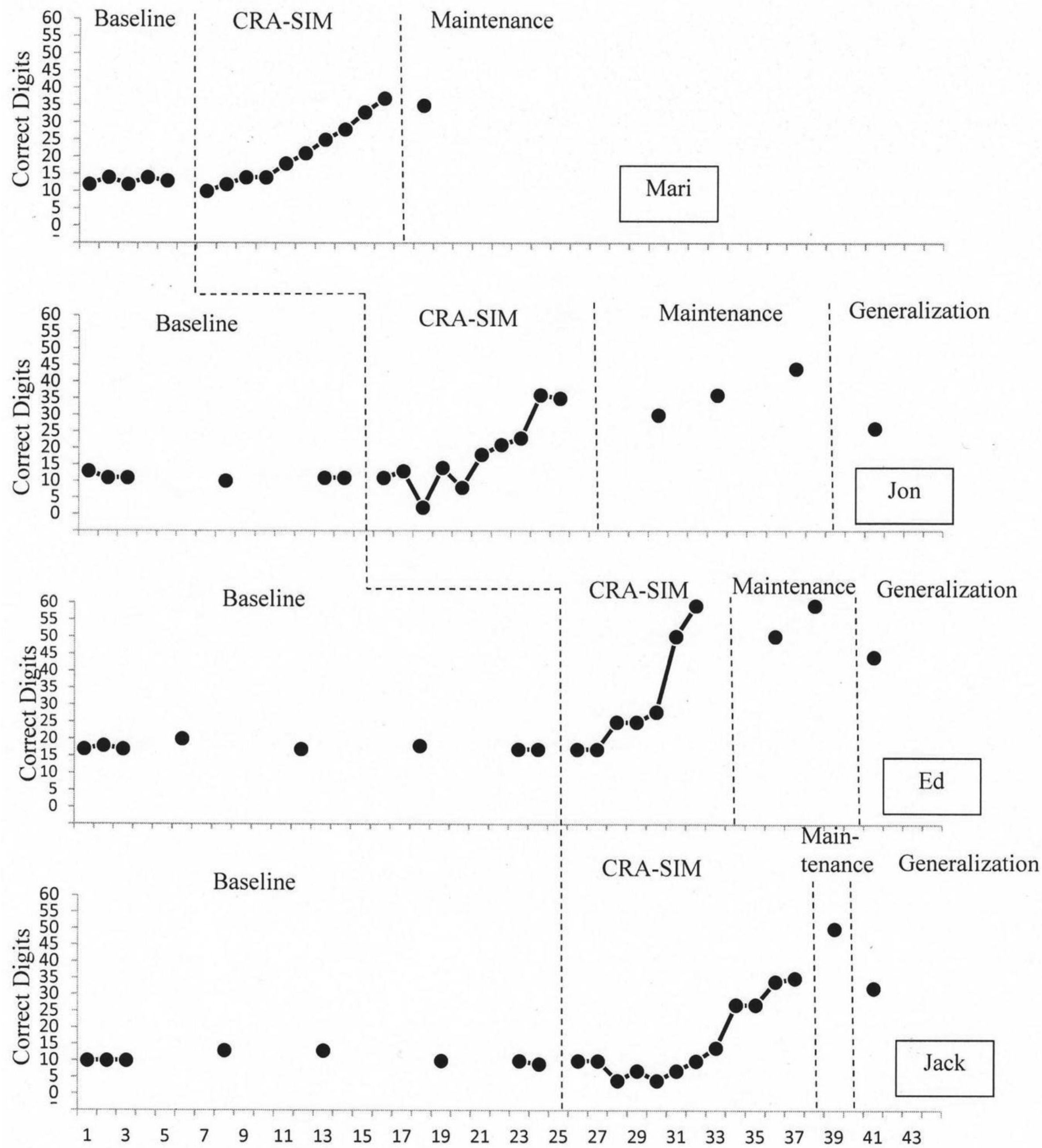


FIGURE 3 Results for Mari, Jon, Ed, and Jack.

Mari's baseline data ranged from 12 to 14 correct digits with a level of 13. Jon's baseline data ranged from 10 to 13 with a level of 11 correct digits. Ed's baseline data ranged from 17 to 20 with a level of 18 correct digits. Jack's baseline ranged from 9 to 13 with a level of 11 correct digits.

Intervention

After an initial decrease in performance, Mari demonstrated an increasing data path that ranged from 10 to 37 correct digits with a level or mean of 21 during intervention and 40 percent overlap between phases. She reached mastery after 11 probes. Mari completed a maintenance probe 1 week after instruction; the researchers administered this probe early because she was moving to another school. Additional maintenance and generalization data were not collected.

Jon's probes ranged from 2 to 36 with a level or mean of 18 during intervention and 40 percent overlap between baseline and intervention. Jon maintained his performance, writing 38 correct digits 2 weeks after instruction ended, 36 correct digits 3 weeks after instruction ended, and 44 digits after 4 weeks of no instruction. Four weeks after instruction, Jon completed a generalization probe and wrote 26 correct digits.

Ed's performance remained similar to baseline in the concrete stage. His performance increased quickly, meeting criterion after the first abstract lesson using numbers only. Ed had seven probes to the criterion with a level or mean of 30 during intervention and 29 percent overlap between baseline and intervention. Ed's data path showed an increasing trend with a range from 17 to 59. He maintained his performance, writing 50 correct digits 2 weeks after instruction and 59 digits after 3 weeks. Ed completed a generalization, writing 44 correct digits.

Jack met criterion after 12 probes with a level of 16 and 58 percent overlap between phases. Two weeks after instruction, he increased performance, writing 50 correct digits. Jack completed a generalization probe 2-weeks after maintenance, writing 32 correct digits.

Effect Size

The researchers calculated Tau-U for each student as well as overall. The Tau-U procedure combines analysis of non-overlapping data points between phases with the intervention phase trend while accounting for any trend within baseline (Parker, Vannest, Davis, & Sauber, 2011). This procedure uses statistical analysis to compare student performance between phases, but also accounts for trends within baseline, which could potentially discount later trends observed within intervention (e.g., if multiplication performance increased prior to intervention, subsequent increases during intervention would be less meaningful, or attributable to the intervention). The use of statistical analysis regarding effect size within single case design allows for analysis and synthesis of multiple single case studies for the purpose of making decisions about practices as evidence-based. No significant trends occurred during baselines. Mari's performance indicated a moderate effect (Tau-U = 0.6). In comparing baseline and intervention phases Jon's performance indicated a moderate effect (Tau-U = 0.6). Ed's performance indicated moderate effect (Tau-U = 0.7). A comparison of Jack's baseline and intervention phases revealed a small effect (Tau-U = 0.1). Overall, the intervention had a moderate effect across all students (Tau-U = 0.5).

Computation Patterns Observed

Mari, Jon, and Jack demonstrated similar patterns across the implementation of the intervention. Each student's performance decreased or overlapped with baseline. After instruction began, the students ceased using their previous erroneous procedures. However, the students did not have the skills or knowledge necessary for correctly solving problems without manipulatives or drawings. New error patterns developed in some cases; however, these errors appeared to reflect some new knowledge of numbers and operations. The students worked slowly, attempting fewer problems, but appeared to be more thoughtful. Over time, fluency improved to 30 correct digits. Patterns of computation across the study are shown in Figure 4.

DISCUSSION

The researchers proposed to investigate the effects of CRA-SIM in teaching multiplication with regrouping to students with SLD. Visual analysis showed a functional relation with increased fluency to criterion at three different points in time with four students. The students demonstrated maintenance of their skills over time when given problems without assistance or other aids such as manipulative objects or pictures. Because Mari moved away, only three students had the opportunity to show generalization. When presented with a near generalization task, two students demonstrated fluency by writing more than 30 correct digits. Jon wrote 26 correct digits, demonstrating some generalized learning.

The calculations of effect size show moderate strength of CRA-SIM. The amount of overlap between baseline and intervention phases affected these findings. The amount of overlap may be viewed as problematic; however, the overall goal of this intervention was fluent computation, a skill developed over time rather than immediately. Immediacy of effect is not necessarily the only hallmark of effective interventions; furthermore, what this intervention may have lacked in immediacy of effect was balanced with the development of fluency and generalization to a similar task. The results of this study are logical based on the design of CRA-SIM instruction. The probes assessed accuracy and efficiency in computation; instruction guided the students toward fluency, with an emphasis on conceptual understanding during the concrete and representational levels of instruction (lessons one through six). It would be expected that fluency would develop after attainment of conceptual understanding. In addition, fluency would logically be demonstrated after instructional practice using numbers only (abstract level). Therefore, even though the results do not show immediate powerful effects, the students' demonstration of learning, maintenance, and generalization should not be discounted. Furthermore, the research design highlights this effect because it involves frequent measures of behavior (daily for this study). Had a different design been implemented, such as one with pre- and postassessments, this learning trajectory would not have been shown; however data regarding learning are important for researchers and practitioners in future implementation. That said, it is important in the future to conduct experimental research studies in which implementation of CRA-SIM is compared to other methods for teaching multiplication with regrouping skills.

The findings are significant because they begin a new line of CRA-SIM research for students with SLD. This study is different from previous CRA-SIM research in that the physical manipulation of large numbers of objects and problem solving procedures were more complex than those required for other operations in the literature (Flores, 2009, 2010; Harris et al., 1995; Mancl et al., 2012; Miller & Kaffar, 2011; Morin & Miller, 1998). Nonetheless, the students completed all levels of instruction without difficulty. A critical instructional component was the place value mat that assisted in

Changes in computation performance for Mari (Bold numerals represent correct digits.)

Baseline	Intervention Concrete Level	Intervention Representational Level	Intervention Abstract Level
^{1}2 6	^{1}3 3	^{1}2 4	^{12}4 5
x 4 3	x 2 4	x 2 4	x 2 5
7 8	**1** 6 **2**	^{1}4 **9 6**	**2 2 5**
		+ **8 0**	+ **9 0 0**
		5 7 6	**1, 1 2 5**

Changes in computation performance for Jon (Bold numerals represent correct digits.)

Baseline	Intervention Concrete Level	Intervention Representational level	Intervention Abstract Level
^{1}5 3	^{1}2 3	^{1}3 3	^{1}5 4
x 1 4	x 2 4	x 2 5	x 2 3
6 **2**	**9 2**	**1 6 5**	^{1}**1 6 2**
	4 **0**	+ **6 6 0**	+ **1 0 8 0**
		7 2 2	**1, 1 4 2**

Changes in computation performance for Ed (Bold numerals represent correct digits.)

Baseline	Intervention Concrete Level	Intervention Representational level	Intervention Abstract Level
3 5	2 3	^{1}3 3	^{1}4 4
x 2 4	x 2 4	x 2 5	x 2 3
6 2**0**	8 1**2**	**1 6 5**	^{1}**1 3 2**
		+ **6 6 0**	+ **8 8 0**
		8 2 2	**1, 0 1 2**

Changes in computation performance for Jack (Bold numerals represent correct digits.)

Baseline	Intervention Concrete Level	Intervention Representational level	Intervention Abstract Level
3 3	^{1}3 3	^{1}5 4	^{2}4 4
x 2 4	x 2 4	x 2 3	x 2 5
6 1**2**	3 **2**	^{1} 9 **2**	^{1}**2 2 0**
	+ **6** 6	+ **1 0 8 0**	+ **8 8 0**
	9 8	**1** 1 9 **2**	**1, 1 0 0**

FIGURE 4 Changes in computation performance across probes for each student.

organizing base-ten blocks and drawings. The mat's utility in the current study is consistent with the findings of other CRA-SIM research involving operations with regrouping (Mancl et al.; Miller & Kaffar).

Limitations and Future Research

Although the CRA-SIM intervention implemented in this project was effective for the participating students, the study was not an experimental study, and does not, therefore, demonstrate the effectiveness of CRA-SIM over and above the effects of additional instruction alone. It is possible that the students improved because they received additional 1:1 instruction on multiplication with regrouping. To demonstrate the effectiveness of the CRA-SIM specifically, it would be important to conduct a randomized control trial study. Nonetheless, our goal was to illustrate the implementation of the CRA-SIM approach for multiplication with regrouping, and to demonstrate in what way it was effective for our participating students.

A second limitation of this study was that instruction was delivered by the researcher. Future research should be conducted in authentic settings in which a special education teacher implements instruction. Although this is a limitation, the method could be implemented by a classroom teacher in a small setting. The researcher, a special education teacher, followed written procedures developed prior to the study which another teacher could follow. Another limitation was the size of instructional groups; future research should investigate the effects of instruction with group sizes that are more realistic for a resource setting. Instructional groups were small based on availability of students, their classroom schedules, and the research design. Other CRA-SIM studies have been conducted with larger groups, and it seems realistic that larger groups would be successful. However, instruction using the concrete objects and drawings is likely to be more successful in group sizes in which the teacher can easily interact with students and monitor their activities. Generalization of findings is another limitation. CRA-SIM was effective for the four students in this study, but additional replication is needed to demonstrate its efficacy for others. Future research should include larger groups of students with varied characteristics.

Implications for Practice

This study provides initial evidence that CRA-SIM is effective supplemental instruction for students with SLD. Students made progress with a small portion of class time and materials readily available in elementary schools. In typical classrooms, mathematics instruction involves differentiation using various instructional models that allow for small group instruction such as station teaching or collaborative approaches. Based on the results of this study, the use of CRA-SIM instruction would be appropriate for students who had not been successful using other methods, but needed an explicit method which scaffolds instruction in the traditional multiplication with regrouping algorithm that emphasizes conceptual understanding. CRA-SIM instruction would also be appropriate for students who make computation errors because they had been introduced to the standard algorithm without firm understanding of numbers and operations. This study allowed for learning aligned with the CCSS (National Governors' Association Center for Best Practices & Council of Chief State School Officers, 2010), demonstrating understanding by representing numbers and operations. CRA-SIM also provided remediation of skills previously taught using general education curriculum materials in the general education and resource setting. Prior to CRA-SIM, the students lacked conceptual knowledge of number systems and operations. After instruction, the students maintained learning over time. CRA-SIM instruction may be an effective method of increasing students' access to CCSS.

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