EDUCATION AND TREATMENT OF CHILDREN Vol. 42, No. 1, 2019

Improvement in Elementary Students' Multiplication Skills and Understanding after Learning Through the Combination of the Concrete-Representational-Abstract Sequence and Strategic Instruction

Margaret M. Flores

Vanessa M. Hinton

Auburn University

Abstract

Recent multiplication with regrouping research shows that the combination of the concrete-representational-abstract (CRA) sequence and the Strategic Instruction Model (SIM) has been effective in several studies. More evidence is needed to demonstrate CRA-SIM's effectiveness across settings and students. Therefore, the purpose of this study was to replicate previous findings in which CRA-SIM led to improved performance of students receiving tertiary interventions. This study extended the research by including problem-solving application in order to align the intervention with current standards for mathematics. Three elementary students receiving tertiary interventions participated in the study. The researchers used a multiple baseline across students design to show a functional relation between CRA-SIM and student performance. In addition to mastery of multiplication with regrouping, students applied their knowledge, discriminating between different operations when solving word problems. The implications and program components that influenced these results will be discussed.

Keywords: mathematics, interventions, elementary

The mathematics standards adopted throughout most of the United States emphasize students' firm conceptual understanding of operations and their relations (Common Core State Standards Initiative [CCSSI], 2010). Within the CCSSI standards, students must represent, model, and explain their computation and problem solving. For students who have difficulties in mathematics and need more intensive interventions, the concrete-representational-abstract sequence (CRA)

Address correspondence to: Margaret M. Flores, Ph.D., BCBA-D, Department of Special Education, Rehabilitation, & Counseling, 2084 Haley Center, Auburn University, AL 36849. E-mail: mflores@auburn.edu

Pages 73–100

and the strategic instruction model (SIM) may provide students with cognitive scaffolding to meet current standards.

CRA-SIM Within Intervention Frameworks

The line of research regarding implementation of CRA and SIM (CRA-SIM) has been conducted with students receiving tertiary interventions (Flores, 2009; Flores, 2010; Flores & Franklin, 2014; Flores, Hinton, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2011). Its classification, as such, is based on the framework described by Lembke, Hampton, and Beyers (2012). When a homogeneous group of students demonstrates a specific skill deficit not remediated through instruction in the general education classroom (i.e., universal intervention) or within small-group instruction, those students receive additional practice opportunities and re-teaching (i.e., targeted intervention). The intensity of intervention using CRA-SIM is increased because the focus of instruction is on one concept and the needs of the instructional group are homogeneous. In addition, instruction within the line of CRA-SIM research was more intensive in its implementation because it occurred in separate settings with a small student-to-teacher ratio; the current study involved a one-to-one student-to-teacher ratio. Each student received explicit instruction and mastered each component of conceptual understanding before progressing to a more advanced level of computation.

Description of CRA and SIM

The CRA sequence provides scaffolding by teaching operations using manipulative objects at the concrete level and drawings at the representation level before teaching operations using numbers only at the abstract level (Hudson & Miller, 2006). Beginning at the concrete level, students use manipulative objects, such as base ten blocks, to solve problems. Manipulatives provide physical models for the operation; the physical representation of the operation assists in translating the mathematical concept into verbal language within instruction. For example, the meaning of the problem 23x4 is shown by obtaining two tens blocks and three ones blocks and then making four groups of each component of 23. The base ten blocks provide a physical model paired with instructional language referring to the multiple groups of two tens and three ones. At the concrete level of CRA, students must make meaning of the numbers that compose 23, rather than conceptualizing them as simply numerals (i.e., two ones and three ones written next to each other). Students must then conceptualize them as two tens and three ones. The representational level of instruction involves drawings where the students draw representations of numbers rather than using base ten blocks (Hudson & Miller, 2006). After computing problems accurately using objects and drawings, the students learn a strategy for remembering the procedural methods associated with completing an operation. SIM is combined with CRA to bridge the gap between the use of physical and visual aids and computation using just numbers (Miller, Stringfellow, Kaffar, Ferreira, & Mancl, 2011). The use of SIM provides students with a set of steps that guide their thinking processes when completing problems. Students use these steps after conceptual understanding has been demonstrated.

The CRA-SIM combination has been used to provide computation interventions for students with and without disabilities at both the elementary and secondary levels. Researchers have shown CRA-SIM to be effective in the area of basic addition, subtraction, multiplication, and division (Mercer & Miller, 1992; Morin & Miller, 1998). Within these studies, students solved basic computation problems using manipulatives, drawings, and a strategy (i.e., discover the sign, read the problem, answer or draw and check, and write the answer [DRAW]). The measure of mastery was percentage of problems correct. Peterson, Mercer, and O'Shea (1988) taught identification of place value using CRA and the FIND (find the columns, insert a T, name the columns, and determine the answer) strategy. CRA-SIM resulted in increased accuracy in addition with regrouping (Miller & Kaffar, 2011) and increased accuracy and fluency in subtraction with regrouping (Flores, 2009; Flores, 2010; Mancl et al., 2012). Flores (2009) combined CRA with the DRAW strategy and Mancl et al. (2012) used a more specific strategy for regrouping (read the problem, examine the ones, note the notes, address the tens, mark the tens, and examine the hundreds and exit with a check [RENAME]). Maccini and Hughes (2000) and Maccini and Ruhl (2000) combined CRA with a strategy to teach algebra concepts (search the word problem, translate the problem, answer the problem, review the solution [STAR]). The focus of this manuscript is multiplication with regrouping which began with an intervention for third grade students and has continued with students with and without disabilities at the elementary level.

CRA-SIM and Multiplication with Regrouping

Flores, Hinton, and Strozier (2014) used CRA-SIM to provide computation interventions to third grade students receiving tertiary interventions within a tiered intervention framework. The students did not have disabilities and received additional daily mathematics instruction during a time period devoted to differentiated instruction and intervention. The researchers provided CRA-SIM interventions across skills: subtraction with regrouping, in which problems required regrouping once (e.g., 342–138); subtraction with regrouping, in which problems required regrouping more than once (e.g., 302–189); and multiplication with regrouping for problems that contained a one-digit multiplier (e.g., 46x4). CRA-SIM multiplication instruction involved learning at the concrete level with base ten blocks and at the representational level using drawings. Next, the students used the RENAME strategy for computing problems. Three students demonstrated mastery of all three skills, including multiplication. However, this study did not explore more complex computation with two-digit multipliers or application with word problems. Additional research was needed to refine instructional materials. The students in this study organized their materials and drawings next to the written problem on a learning sheet; the researchers questioned whether additional tools for organization would enhance learning.

Flores, Schweck, and Hinton (2014) investigated the use of CRA-SIM for teaching multiplication with regrouping in which problems included two-digit multipliers. The students in this study were older, fifth grade students with specific learning disabilities who received special education services in the area of mathematics. The researchers used the same instructional materials with the addition of a laminated place value mat on which students organized base ten blocks during concrete-level instruction. The students also used the mat to organize representational-level drawings. Using a multiple probe across students design, the researchers demonstrated a functional relation between CRA-SIM instruction and changes in students' computational fluency in solving multiplication with two-digit multipliers. CRA-SIM, with the addition of a place value mat, was effective for teaching more complex operations.

Both of the previous studies using CRA-SIM to teach multiplication with regrouping were implemented by the researchers rather than in an authentic setting by a teacher. Flores and Franklin (2014) investigated the effects of CRA-SIM's implementation by a teacher in an intervention setting. Using an instructional manual, place value mats, base ten blocks, and student learning sheets, a general education teacher used CRA-SIM to teach multiplication with regrouping and two-digit multipliers to fourth grade students enrolled in an after-school tertiary intervention program. This pilot study of instructional materials by another teacher showed that students made significant growth based on assessments completed before and after the study.

This line of research has shown that the use of CRA-SIM to teach multiplication with regrouping has led to improved computation for students receiving tiered interventions and students with disabilities. In order to show that CRA-SIM is an evidence-based practice, further replication of the intervention with positive results is needed. In addition, the presentation and organization of student materials could be further enhanced by situating computation within real world problems. For example, previous studies involved presenting students with problems using numbers only. However, understanding the operation, its relation to other operations, and its application may be better presented through word problems rather than just equations. Therefore, the purpose of this study was to replicate the findings of previous research in which CRA-SIM intervention improved the computation performance of students receiving tertiary interventions. In addition, this study extended the research by changing CRA-SIM to include application and problem solving within instructional lessons so that the intervention aligns with current standards for mathematics.

Method

Setting

The study took place in an elementary school located in a rural community near a large state university in the southeastern United States. Instruction occurred during a portion of an afterschool care program. The setting for instruction was a classroom that was used for small-group instruction during the regular school day but was available for use at the end of the school day. The first author, a certified special education teacher, provided the intervention. Implementation of the intervention by the students' teacher or school staff member during the regular school day was not feasible.

Participants

The students who participated, Toni, Tom, and Tina, were enrolled in the third grade and received tertiary mathematics interventions within a failure prevention framework. None of the students had identified disabilities. In order to participate, students had to meet the following criteria: (a) parent permission to participate in research, (b) proficiency in addition and subtraction with regrouping as defined as writing at least 20 correct digits within two minutes, (c) fluency in basic multiplication as defined as writing 30 correct digits within one minute, and (d) lack of proficiency in multiplication with regrouping as defined as less than 25% of problems completed correctly on a multiplication with regrouping probe. At the point of intervention, the students had received instruction in multiplication with regrouping

Table 1
Student Demographics

Student	Age	Grade	Cultural Background	Cognitive Ability (IQ)[a]	Mathematics Computation Achievement[b]
Toni	9	3	White	105	88
Tom	10	3	White	88	86
Tina	9	3	African American	91	82

a. Standard Score *Kaufman Brief Intelligence Test 2nd Edition* (Kaufman & Kaufman, 2004)

b. Standard Score for Operations *Key Math 3 Diagnostic Assessment* (Connolly, 2007)

in their general education classroom and as part of targeted instruction, or tier two intervention. Tier two intervention for these students involved small-group instruction implemented during a designated time period in which all students in the school received either enrichment or remediation. Tier two interventions involved repeated lessons using curriculum materials previously used in students' general education instruction, but within a smaller group, over the course of a nine-week period. Tier two instruction did not result in improvement of the students' mathematics grades and the school problem-solving team determined that they had not made adequate progress based on monthly assessments. The authors were not given access to the students' assessment records, so specific progress monitoring procedures cannot be discussed. If the students had not participated in this study, they would have received tertiary interventions that would involve smaller student-to-teacher ratios, repeated instruction, and practice related to focused concepts such as multiplication with regrouping. The students' mathematics instruction within the general education classroom at the time of the study was in division and fractions. No instruction in multiplication with regrouping occurred except within the study's intervention. Student characteristics are summarized in Table 1.

Materials

Assessment materials. There were three types of assessments: repeated computation probes, a problem-solving assessment, and an interview related to computation procedures. The purpose of the study was to investigate students' computation performance regarding multiplication with regrouping, which was taught within the context of application using words and word problems. The materials for assessment included quantitative measures of computation used for repeated assessment to show change in computation. Since students learned computation within the context of the operation's application, the researchers administered one descriptive measure after the intervention for the purpose of assessing the students' learning related to application and understanding of the operation. There was one descriptive measure of students' procedural understanding in the form of an interview related to explanation of the computation procedures.

The researchers measured students' computation progress using multiplication probes. These were sheets of paper with twenty problems in which the multiplicand (i.e., top number) was a two-digit number and the multiplier (i.e., bottom number) was a one-digit number. Each problem required regrouping. There were five different probes, each containing different problems. The probes used for this study had been used in previous research in which reliability scores were obtained (Flores, Schweck, & Hinton, 2014). The results from internal consistency tests showed that Cronbach's α coefficient of $r = 0.73$.

The researchers obtained two scores from each computation probe: the number of correct digits and the percentage of completed computation problems that were correct. The number of correct digits provided a measure of progress and fluency; the percentage of correctly completed problems ensured that the student completed problems accurately. Both measures were used in order to avoid student products where all problems were completed partially correctly, showing many correctly written digits, but lacking mastery of the operation with few or no correct products (e.g., the problems 26x5=1030 or 34x4=166 include two correct digits, but the product and the process used to compute the product are incorrect).

The researchers measured students' progress regarding application using one probe with six word problems that were given after computation instruction. The word problems involved one step and required addition, subtraction, or multiplication with regrouping. The problems were written at a second-grade level to ensure that the students could decode the words within each problem. The purpose of this probe was to assess the students' progress related to application of the operation. The data obtained from this assessment showed that the students could discriminate between operations, understanding the difference between multiplication, addition, and subtraction. Discrimination was included within instruction and this assessment provided information regarding student learning beyond just computation skills.

The final descriptive measure involved an interview with each student using a sheet with one multiplication with regrouping problem written vertically that had not been completed. Before and after

1) There are 15 bags of candy, 4 candies in each bag. How many pieces of candy all together?	2) There are 12 students and each one has 5 pencils. How many pencils in all?	3) There are 5 classes with 23 students in each class. How many total students?
_____ groups of ______	______ groups of _______	_____ groups of ______
_____ x _______	______ x _______	___ x ___ or ___ x ___
1 5 x 4	1 2 x 5	2 3 x 5

Figure 1. Examples of items used for concrete instruction.

the study, the researcher asked the student to talk out loud and explain the computation process. The purpose of this interview was to gather information about the students' understanding of numbers, operations, as well as how and why one would use the computation procedures. The data obtained included word-for-word student descriptions of the numbers manipulated in the computation process as well as the computational procedures used to solve the problem.

Instructional materials. The instructional materials consisted of an instructional manual with lesson guidelines and suggested scripts to guide teacher behaviors and language. The student materials consisted of ten lessons: three for concrete instruction, three for representational instruction, one lesson to teach the RENAME strategy, and three abstract lessons. Additional abstract instructional lessons were available in case students did not reach the criteria for mastery after ten lessons. Concrete lessons involved problems using short scenarios and their translation into mathematics problems. Examples of items used for concrete instruction are in Figure 1.

Concrete-level instruction included a laminated place value mat and a set of base ten blocks. Student learning sheets for representational instruction presented problems using short scenarios and a place value table printed next to the problem. Examples of items used for representational instruction are in Figure 2.

Abstract-level instruction included problems written using numbers only as well as word problems that required multiplication, addition, or subtraction. In addition to demonstrating computation using numbers only, students discriminated among operations in order to correctly solve word problems. Multiple word problems were presented and taught so that students would not be encouraged to solve all word problems using multiplication. Students would use

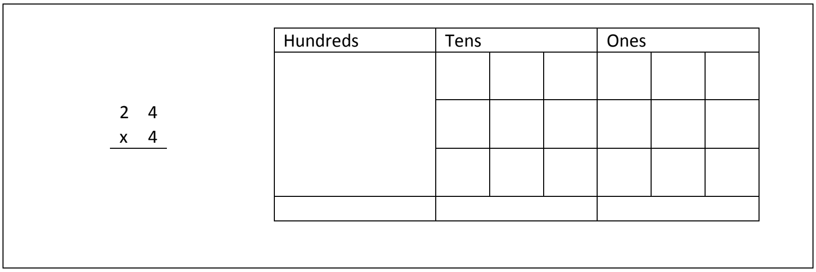


Figure 2. Example of item used for representational instruction.

what they learned about the operation to discriminate its application from that of other operations.

Examples of items used for abstract instruction are in Figure 3.

Procedures

Assessment procedures. The quantitative assessment materials used to measure students' computation progress were given during both baseline and instructional phases. The researcher gave the student a probe and said that he/she had two minutes to complete as many problems as he/she could. The researcher started a timer and at the end of two minutes, collected the probe. Probes administered during the intervention phase were given prior to instructional lessons with the intention of measuring skills retained by the student from the previous day's lesson.

1) 3 5 x 4	2) 4 3 x 6	3) There are 24 boxes with 4 toys in each box. How many toys in all? Write and solve.
4) There are 16 apples 15 oranges in the bowl. How fruits are in the bowl? Write and solve.		5) There are 65 pieces of candy, 45 are chocolate. How many are not chocolate? Write and solve.

Figure 3. Examples of items used for abstract instruction.

After all instructional lessons were taught, the students completed one application problem-solving probe. The procedures for administration involved giving the probe to the student, asking her/him to complete the problems, and informing the student that there was no time limit. After providing instructions, the researcher asked the student to read the problems aloud to confirm that each student could decode the words within each problem.

Another descriptive measure involved collecting information related to the students' understanding of the operation by interviewing students about how to solve multiplication with regrouping problems. The researcher presented each student with a multiplication computation problem. The researcher asked the student to describe and show how to compute the problem. The researcher asked what each numeral in the problem meant or represented. The students' answers were written word for word.

Prior to the study, each student responded that they did not know how to do those kinds of problems. Their responses were: "I do not know," "I cannot do that," "We have not done that." None of the students provided additional information when asked to guess or attempt. When asked about the numbers shown in the problem, the students identified each as a numeral. When asked about the number 26, the students pointed to the numeral in the tens place and identified it as two. The researcher probed further by asking what was meant by the answer "two" and the students each repeated their answer of "two," rather than twenty or two tens.

Instructional procedures. One of the researchers served as the instructor and worked with each student individually. The researcher provided instruction four days a week for 20 minutes each day for three weeks. Different lengths are attributed to variance in student progress. For example, one student mastered the skill within three weeks over the course of ten lessons, another student mastered the skill at the three-week point with eleven lessons, and the third student mastered the skill in three and a half weeks with fourteen lessons.

The instructor used the explicit instructional sequence throughout each level of instruction. The instructor provided: (a) an advance organizer, telling the student what would happen over the course of the lesson; (b) demonstration of problem solving through physical actions as well as thinking aloud; (c) guided problem solving, in which the instructor and student took turns trading tasks back and forth with prompting; (d) independent practice, in which the student solved problems without assistance; and (e) a post organizer, in which the instructor briefly reviewed the lesson events.

Instruction began at the concrete level with the presentation of sentences that were translated from words into multiplication equations. Rather than teaching multiplication with regrouping using just equations, sentences were used to show the application of the multiplication concept, repeated addition, or adding groups that each included the same amount. The translation process involved scaffolding. An example sentence was, *There are 24 boxes of books and each box has 6 books. How many books in all?* The information from these sentences was used to tell how many groups and how many things were in each group. On the lesson sheet, there was a place to note this. Once the number of groups and the number of objects in each group were established, a multiplication equation was written using numbers and symbols (i.e., ___ x ___ was filled in with 24 x 6). Finally, the student and teacher solved the multiplication problem, written vertically. With the assistance of base ten blocks and a multiplication mat, the student and teacher solved the vertically written multiplication equation.

Using base ten blocks, the multiplicand was decomposed (e.g., 24 was two tens and four ones) and blocks were placed in the appropriate columns on the place value mat. Using the blocks and mat, problem solving according to the traditional algorithm began. Starting with the ones, the instructor and student made groups of blocks according to the multiplier (six groups of four ones). Regrouping occurred according to the following rule: *if there are ten or more, go next door.* In this problem example, six groups of four resulted in 24, a number that was more than ten. Ones blocks were exchanged for tens blocks and the tens blocks were placed in the tens column of the mat. On the written problem, the number two was written above the numeral in the tens place of the multiplicand to note regrouping. Next, the number of ones blocks that remained on the mat was noted on the written problem (four blocks remained in the ones column of the mat, so the numeral four was written in the ones place of the problem).

According to the traditional algorithm, the numeral in the tens place of the multiplicand was multiplied by the multiplier. To complete this process, the instructor and student made groups of tens blocks according to the multiplier (e.g., six groups of two tens). After making groups, the instructor and the student counted the number of tens and applied the rule about *ten or more* (the problem example involved 12 tens). Tens blocks were exchanged for a hundreds block and the hundreds block was placed in the hundreds column of the mat. The instructor and student marked the problem according to the blocks on the mat. Before moving to another problem, the instructor and student compared the place value mat with the numbers in the written problem. If the answer for the written problem was 144, the place value mat had one hundreds block, four tens blocks, and four ones blocks in the appropriate columns.

Representational-level instruction involved the use of the same written sentences. However, the translation of the sentence into a mathematical equation was verbal without the written prompts used at the concrete level (e.g., ___ groups of ___ and ___x___). Rather than using base ten blocks, the instructor and student drew representations of the base ten blocks. Hundreds were drawn as squares, tens were drawn as vertical lines, and ones were drawn as short tallies written on a horizontal line. The instructor and student drew on a replica of the place value mat that was printed on the student learning sheet. Beginning with the ones column of the written problem, the instructor and student drew groups of ones according to the multiplier (six groups of four ones). For this problem example, there were ten or more. Two groups of ten tallies were circled and long lines were drawn horizontally above the tens place. Regrouping was noted on the written problem by writing a small numeral above the tens place. Using the number of remaining ones (i.e., un-circled short tallies), the teacher and student wrote the number on the written problem in the ones place. According to the multiplication algorithm, after multiplying numbers in the ones place, the next step is multiplying by the numeral in the tens place of the multiplicand. The instructor and student drew groups of tens according to the multiplier (i.e., six groups of two tens). After drawing the groups, the instructor and the student counted the number of tens and applied the rule about *ten or more*. Since there were ten or more in this example problem, 10 long vertical lines were circled and a square representing one hundred was drawn in the hundreds column of the table. The instructor and student marked the problem according to the drawings. Before moving to another problem, the instructor and student compared the drawings with the numbers in the written problem. The CRA process for solving multiplication problems with regrouping at the concrete and representational level is shown through pictures in Figure 4.

After representational instruction, the seventh lesson involved learning the RENAME strategy. The instructor solved a problem using RENAME, demonstrating the strategy's use. The student recited the strategy steps as they were written. The instructor showed the first letter of each step and the student recited the step. This continued until the student could recite the strategy when given just the mnemonic device.

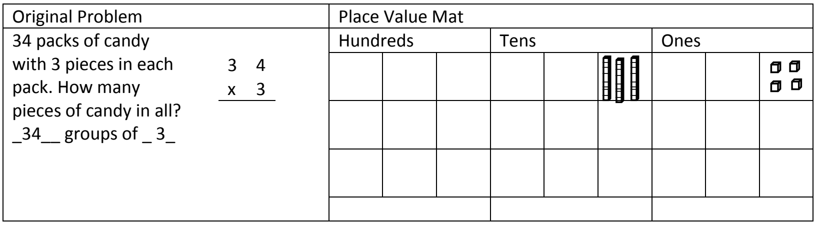
Instruction at the abstract level began once the student could recite each step of the RENAME strategy. Abstract instruction involved computation of problems using only the strategy. Problems within abstract lessons were presented two ways: written numerals only in the form of vertically written equations and one-step word problems that required addition, subtraction, or multiplication with regrouping. The instructor and student used the RENAME strategy to solve multiplication problems. Solving word problems involved discrimination among the operations and problem solving using RENAME. For example, the instructor and student thought aloud and talked about what was happening within the word problem, whether numbers were combined to find the answer (e.g., addition or multiplication) or separated to find the answer (e.g., subtraction). If the numbers were combined to find the answer, the teacher and the student determined whether there were groups, each with the same amount (e.g., multiplication) or whether there were groups with different amounts combined (e.g., addition). Since the students were proficient in addition and subtraction prior to the study, computational procedures using RENAME for addition and subtraction were not included in instruction. Once the teacher and student correctly identified the operation needed to solve word problems, students solved addition and subtraction problems without the instructor's assistance. Multiplication problems were solved using the RENAME strategy. Problems involving division were not included because, at the beginning of the study, division instruction had not been completed within their general education mathematics classroom.

As stated in the materials section, there were ten lessons total and extra lessons were included at the abstract level. Toni completed the intervention and reached criteria after ten lessons. Tom completed ten lessons plus one extra lesson at the abstract level. Tina completed ten lessons plus four additional abstract lessons before reaching the criteria for mastery.

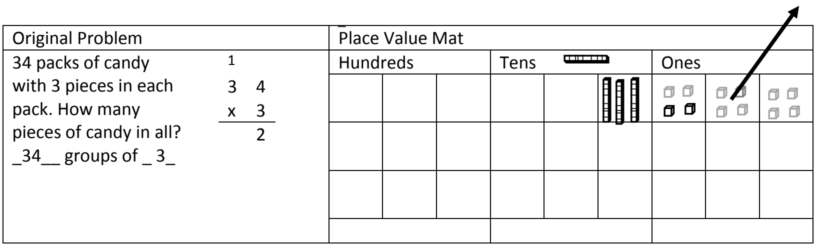
Training

Prior to the collection of treatment fidelity data, the observers received training in the identification of examples and non-examples of implementation behaviors. The observer watched lesson implementation, indicating whether behaviors were observed or not observed. Training was finished once identification with 100% accuracy occurred across three sessions. Training with respect to scoring probes involved scoring multiplication probes used in previous studies and checking for accuracy in counting digits correct as well as counting

Step 1 Concrete: Decompose 34, making three tens (30) and four ones (4).



Step 2 Concrete: Multiply four and three, making three groups of four. If there are ten or more, regroup by removing ten ones blocks and replacing them with a tens block in the tens column. Write the numeral in the ones place and note regrouping in tens place.



Step 3 Concrete: Multiply thirty and three, making three groups of thirty. Add the additional ten to the product. If there are ten or more, regroup by removing ten blocks and replacing them with a hundreds block in the hundreds column. Write the numerals in the tens and hundreds places.

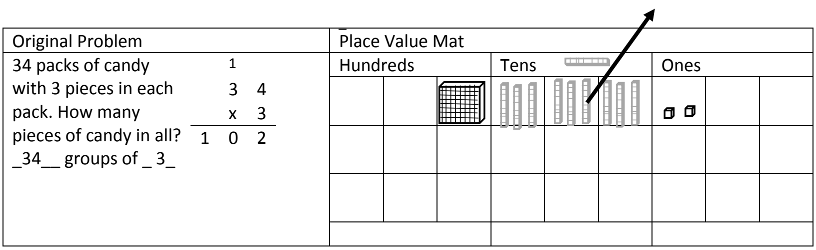
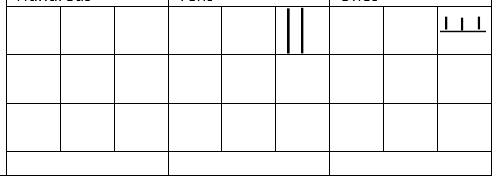


Figure 4. Problem solving processes for concrete and representational instruction.

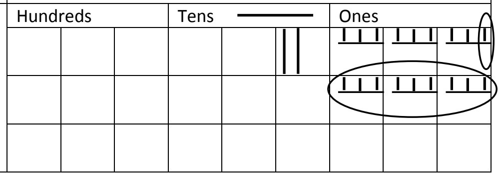
Step 1 Representational: Decompose 23, making two tens (20) and three ones (3).

Original Problem		Place Value Mat		
23 boxes of books with 6 books in each box. How many books in all? _23__ groups of _ 6_	2 3 x 6	Hundreds	Tens	Ones



Step 2 Representational: Multiply six and three, making six groups of three. If there are ten or more, regroup by circling the ones and drawing another ten in the tens column. Write the numeral in the ones place and note regrouping in tens place.

Original Problem		Place Value Mat		
23 boxes of books with 6 books in each box. How many books in all? _23__ groups of _ 6_	1 2 3 x 6 8	Hundreds	Tens	Ones



Step 3 Representational: Multiply twenty and six, making six groups of twenty. Add the additional ten to the product. If there are ten or more, regroup circling the tens and drawing one hundred in the hundreds column. Write the numerals in the tens and hundreds places.

Original Problem		Place Value Mat		
23 boxes of books with 6 books in each box. How many books in all? _23__ groups of _ 6_	1 2 3 x 6 1 3 8	Hundreds	Tens	Ones

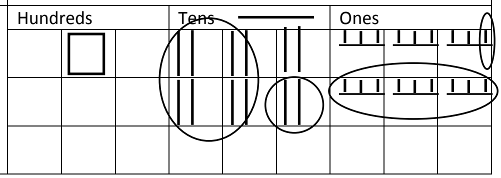


Figure 4. (continued)

the number of answers correct. Training was finished when a set of 10 probes was scored accurately.

Treatment Fidelity and Inter-Observer Agreement

The researchers measured treatment fidelity through direct observations and completion of a checklist of instructor behaviors. Treatment fidelity was completed three out of four days each week (i.e., 75% of the lessons) exceeding recommendations for high quality single case design (Poling, Methot, & LaSage, 1995). The second author and another trained observer completed the checklist and indicated whether behaviors were observed or not observed. Treatment integrity was calculated as 100% with 100% inter-observer agreement. The items from the checklist are included in Table 2.

The assessment probes were scored by two researchers independently and the researchers calculated inter-observer agreement by adding the number of item agreements and dividing that sum by the sum of item agreements and disagreements (Poling, Methot, & LaSage, 1995). All baseline and intervention probes were assessed for inter-observer agreement (100%). The inter-observer agreement for Toni's probes was 99% for correct digits (i.e., 314 digits with agreement and 317 total agreements and disagreements). There were three probes in which agreement was discrepant by one digit. Within Toni's individual probes, accuracy in identifying correct digits ranged from 96%–100%. There was 100% agreement for problems correct. The inter-observer agreement for Tom's probes was 98% for digits correct (i.e., 247 digits with agreement and 252 total agreements and disagreements) and 100% for problems correct. There were four probes in which there were discrepancies in the number of digits correct; three probes differed by one digit and one probe differed by two digits. Within individual probes for Tom, accuracy in identifying correct digits ranged from 95%–100%. The inter-observer agreement for Tina's probes was 98% for digits correct (i.e., 247 digits with agreement and 252 total agreements and disagreements) and 100% for problems correct. There were three probes with discrepancies in digits correct; two probes differed by two digits and one probe differed by one digit. Within individual probes for Tina, accuracy in identifying correct digits ranged from 94%–100%.

Social Validity

Social validity data were collected using a student survey created by the researchers before and after the study. The written items asked whether students (a) could multiply large numbers, (b) wanted to learn a new way to multiply, and (c) wanted to improve their multiplication skills. All of the students wrote that they did not know how to complete multiplication problems with large numbers. These student answers were confirmed with the baseline probes. Tom and Tina completed none of the problems during the first three baseline sessions and said that they could not attempt the problems because they did not know how. All of the students indicated that they would like to know how to complete problems and improve their performance.

After the study, the students completed written questions that asked (a) whether they could multiply large numbers, (b) whether multiplication was easier, and (c) what they liked about the intervention. Each of the students reported that computation was easy after the intervention and that they liked CRA instruction. Tina wrote that she preferred solving problems using base ten blocks and Tom

Table 2
Treatment Fidelity Checklist Items

Items Used to Rate Fidelity of Treatment
Instructor gives student a blank probe sheet and instructs him/her to complete as many problems as he/she can.
Instructor uses a timing device for timed probes. After two minutes, instructor collects probes.
All materials ready prior to lesson.
Provides an advance organizer, tells the student what he/she will be doing and why.
Paraphrases suggested script within CRA materials.
Teacher demonstrations are accurate according to program.
Accurately uses CRA procedures throughout each portion of the lesson.
Engages students in instruction during demonstration and guided practice by prompting their participation, asking questions, etc.
Maintains eye contact with students during the majority of lesson.
Uses smooth phrasing throughout all parts of the lesson.
Is enthusiastic when teaching (tone of voice is expressive and natural, loud enough to be heard, but does not interfere with other classroom activities).
During independent practice, instructs the students to solve problems without guidance.
Provides verbal prompts if the student has difficulty.
Monitors the students' work while they solve problems independently. Does not offer answers.
Closes with a positive statement about the student's performance in the feedback process, reviews lesson, and mentions future lesson and expectations.

indicated that he preferred using drawings more than blocks. Toni reported that she liked solving problems using numbers only and liked completing increasing numbers of problems correctly (i.e., scores on probes increased).

A researcher interviewed the students' teacher before the study and asked if the students needed intervention in the area of multiplication with regrouping. The teacher reported that all of the students needed intervention. After the study, the teacher was asked if and how the students' skills improved. The teacher reported that students made more positive comments about mathematics. The teacher reported that the students showed improvement in their multiplication with regrouping skills and problem-solving skills on end-of-the-year benchmark assessments.

Research Design

The researchers investigated the relation between CRA-SIM and regrouping performance using a multiple probe across students design. Baseline data were collected until the data path was stable, as defined as the last four data points prior to intervention having no more than 20% variation from their mean. The first student moved from baseline to intervention when baseline data were stable and continued until the criteria for mastery were reached, writing at least 30 correct digits within two-minutes with 100% of problems completed correctly, meaning that the answers were correct.

The criteria included 30 correct digits because that is the fluency standard for third grade students (Hosp, Hosp, & Howell, 2007). The second student began intervention when the first student wrote at least 20 correct digits within two-minutes and 100% of the problems completed were correct. This student continued the intervention until the criteria for mastery were met. The third student moved from baseline to intervention when the baseline data path was stable, and the second student wrote at least 20 correct digits within two-minutes and 100% of the problems completed were correct. The third student continued in the intervention until the criteria for mastery were met. After the criteria for computation were met, each student completed one untimed problem-solving probe and an interview regarding their computation procedures. The researchers gave a maintenance computation probe two weeks after the study.

The data were inspected visually to investigate the effects of CRA-SIM on multiplication computation performance between baseline and intervention phases. The researchers noted changes based on the level of data paths, the number of data points to the criteria for mastery (30 correct digits and 100% of attempted problems correct), and the percentage of non-overlapping data points (PND). The researchers calculated PND by counting the number of data points that were outside the range of the baseline data. This number was divided by the total number of data points gathered during intervention and multiplied by 100 (Scruggs & Mastropieri, 2013). In addition to PND, the researchers calculated Tau-U, a statistical analysis in which non-overlapping data points, analysis of trend in intervention, and the lack of trend in baseline are combined to generate an effect size in which scores above 0.8 are strong or large, scores of 0.5 are moderate, and scores less than 0.2 are small or weak (Parker, Vannest, Davis, & Sauber, 2011). The Tau-U statistic was used in addition to PND, because PND does not take trend into account; it is important that the baseline data do not show a trend similar to the data in intervention. In addition, this calculation of effect size includes intervention trend in addition to lack of overlap, providing a more comprehensive evaluation of the data.

Results

The graphs depicting the students' results are in Figure 5.

Toni

Toni's mean score for the baseline phase or level was 8.5 correct digits (*SD* = 1.0). Zero percent of the completed computation problems were correct, meaning none of the answers to the equations computed were correct. Both data paths (correct digits and percent correct) were stable in trend. There was an immediate change in the number of digits written between baseline and intervention, but no change in the percentage of problems correct upon phase change. For the intervention phase, the level (mean) of the data path for correct digits was 31.4 with data points ranging from 19 to 44 (*SD* = 10.76). The level (mean) of the data path for percentage of completed problems correct was 69% with data points ranging from zero to 100%. There were nine probes completed before Toni met the criteria for mastery (i.e., 30 correct digits with 100% of problems completed correctly). The PND for correct digits and percent correct were 0% and 20% respectively. Two weeks after instruction ended, Toni demonstrated maintenance by writing 43 correct digits with 100% of the answers correct.

Tom

Tom's mean score for the baseline phase or level was 3.1 correct digits (*SD* = 2.97). He completed 0% of computation problems correctly. Both data paths (correct digits and percent correct) were stable in trend. There was an immediate change in the number of digits written between baseline and intervention, but no change in the percentage of problems correct at phase change. The level (mean) of the data path for correct digits was 23 with data points ranging from 9 to 37 (*SD* = 10.59). The level (mean) of the data path for percentage of completed problems correct was 59% with data points ranging from zero to 100%. There were ten probes completed before Tom met the criteria for mastery (30 correct digits with 100% of problems completed correctly). The PND for correct digits and percent correct were 0% and 30% respectively. Two weeks after instruction ended, Tom maintained his performance by writing 33 correct digits with 100% accuracy.

Tina

Tina's mean performance during the baseline phase (level) was 3.4 correct digits (*SD* = 2.95). She completed 0% of computation problems correctly. Both data paths (correct digits and percent correct) were stable in trend. At phase change from baseline to intervention, Tina's data were similar; the correct digits increased slowly, and the percentage correct remained the same until there was a large increase at the fourth data point in the path. The level (mean) of the data path for correct digits was 30.7 with data points ranging from 7 to 48 (*SD* = 13.46). The level (mean) of the data path for percentage of completed problems correct was 68% with data points ranging from zero to 100%. There were 13 probes completed before Tina met the criteria for mastery (i.e., 30 correct digits with 100% of problems completed correctly). The PND for correct digits and percent correct were 8% and 23% respectively. Two weeks after instruction ended, Tina maintained her performance by writing 48 correct digits with 100% accuracy.

Effect Size

The researchers calculated Tau-U for each student. There were no significant trends for any of the students within baseline phases. In comparing Toni's baseline and intervention phases for correct digits, a strong effect was indicated (Tau-U = 1.0). For Tom, there was a strong effect indicated for correct digits between baseline and intervention phases (Tau-U = 1.0). In comparing Tina's baseline and intervention digits correct data, a strong effect was indicated (Tau-U = 0.98). The researchers found an overall effect for the study with regard to correct digits, finding a strong effect across all students (Tau-U = 0.99). With regard to effect size calculations for the percentage of completed problems correct, the effects were moderately strong. For

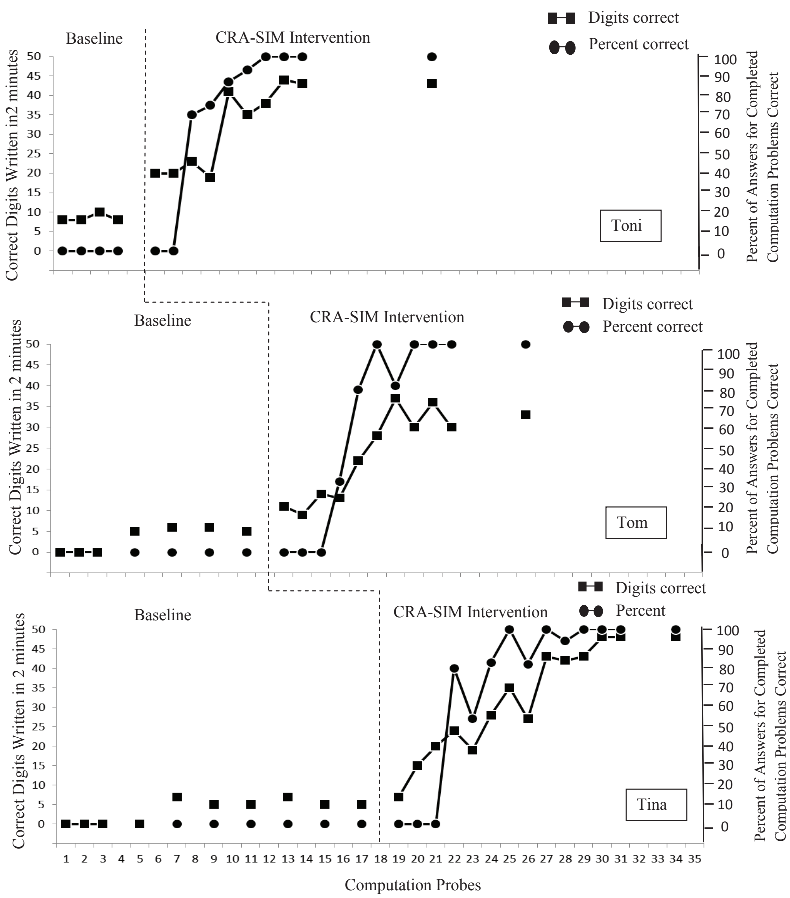


Figure 5. Results for Toni, Tom, and Tina regarding computational probes.

the comparison between baseline and intervention phases for percentage of answers correct, a moderately strong effect was indicated for Toni (Tau-U = 0.78), Tom (Tau-U = 0.70), and Tina (Tau-U = 0.77). The researchers found the overall effect with regard to percentage of problems correct to be moderately strong (Tau-U = 0.75).

Descriptive Measures

After the students completed the last abstract lesson, the researchers administered one untimed assessment of their problem-solving performance in order to assess application of the operation. The problems on the probe involved multiplication, addition, and subtraction. The probe required that students apply their new knowledge of multiplication to word problems as well as discrimination between operations. Toni, Tom, and Tina scored 100% on the probe.

In order to assess the students' approach and understanding of the computation involved in multiplication with regrouping, the researchers asked students to describe their computation before and after the study. The researchers presented students with a multiplication with regrouping problem (i.e., written using numerals) and asked how they would solve the problem. The students' answers were written word for word by a researcher. As described previously in the methods section, the students demonstrated poor understanding of numbers and the multiplication operation before the intervention. After the study, in contrast to their behavior prior to the study, the students readily answered questions about the problem. When asked about the numbers shown in the problem, the students identified numbers based on their value (e.g., 3 tens or thirty and 4 ones or four). The students solved the problem 34x5 by multiplying the numbers in the ones place, telling that four groups of five made twenty. The students explained that there were no ones in the number twenty, but there were two tens. The students marked the ones place with a zero and explained that the two tens were grouped with the tens in the tens column, writing the numeral two above the tens column. The students explained that the number thirty should be multiplied by five. When asked what that meant, students stated that there would be five groups of thirty or five groups of three tens. They added the two tens that had been grouped in the tens column. Each of the students described the numbers in terms of their value, writing the numerals one and seven in the answer, but telling that it was one hundred and seven tens or one hundred and seventy.

Discussion

The purpose of this study was to replicate the findings of previous CRA-SIM multiplication research and extend the research by using materials that encouraged application and problem solving. All three of the students improved their computation performance and achieved fluency in computation of multiplication with regrouping. This is consistent with previous CRA-SIM research related to multiplication with regrouping (Flores, Hinton, et al., 2014; Flores, Schweck, et al., 2014; Flores & Franklin, 2014). The current study differs from previous research in its collection of student computation data with both the measure of correctly written digits and problems computed with an accurate answer. Previous research has included one of these measures, but not both. The current research sought to capture students' incremental progress with correct digits as well as true fluency which includes accuracy and quickness; writing many digits is important, but accurate solutions are critical as well. As the graphs show, the two measures provide a more realistic demonstration of student progress. Even though students wrote more correct digits after intervention began, as shown by no overlapping data points, problems were not immediately solved correctly, as shown with overlap in data points during the beginning of the intervention. At the concrete level, although students solved problems using base ten blocks within lessons, they could not solve problems using numbers only on probes. These effects are consistent with previous research in which accuracy did not improve until the completion of representational or abstract instruction (Flores, Hinton, et al., 2014; Flores, Schweck, et al., 2014).

The students in the current study also demonstrated increased conceptual understanding and skills related to explanation of the operation as measured by their verbal descriptions of the operation during interviews. The interviews showed that students discussed the meaning of the operation, combining numbers together, specifically combining groups with the same amount in each group. The students' verbal descriptions showed that they understood multiplication, making multiple groups of each component of the multiplicand (e.g., the tens and ones within the top number). It is possible that students had some understanding of this concept, but CRA-SIM instruction assisted students in articulating their understanding with words.

In addition to showing improved abilities to articulate their computation, the students demonstrated that they could apply their computational skills within word problems. On one probe administered after the intervention, each student discriminated between situations in which multiplication was appropriate and inappropriate when given word problems that required addition, subtraction, or multiplication. The CRA-SIM intervention within this study extended previous research by including an application probe. In addition, the research was extended by including application activities throughout all lessons and word problems specifically within abstract lessons. The abstract-level lessons included discussion about word problems, specifically the thinking processes involved in discriminating among operations. After the study, students showed that they maintained their skills related to discrimination between operations.

Limitations and Future Research

This study's implementation is limited since the instructor was a researcher. The researcher was a certified special education teacher who provided the intervention in a classroom setting; however, implementation would have been more realistic if the researcher had given the program to a teacher within the students' school for implementation during a regularly scheduled intervention time. The high treatment integrity was influenced by the current type of implementation. Future research should include professional development for a classroom teacher and authentic implementation by that teacher. This would provide better information related to the feasibility and effectiveness of the intervention in real life settings.

Another limitation related to the researcher as instructor is related to social validity. The researcher was present when the students completed the written survey. This may have influenced positive student responses.

The nature of the research design—multiple probe across students—limits the study with regard to generalization as well as its effects compared to other interventions. The effects of the CRA-SIM intervention were shown for a small group of students. Within single case design, generalization is shown with replication of results across studies (Horner et al., 2005). This study replicated previous findings using the strategy, but additional research is needed to draw conclusions regarding generalization. With regard to comparison, it is not known whether CRA-SIM is as or more effective than other computation interventions. The students in this study had previous experience with multiplication with regrouping within their general education setting without success, but the experimental design did not allow for information regarding direct comparison between intervention methods. This limitation can be addressed by future research in which groups of students receive either CRA-SIM or another intervention and students' progress within each intervention is analyzed using statistical methods.

Implications and Conclusions

The current study's findings are significant in that they demonstrated the effectiveness of CRA-SIM for three students receiving tertiary interventions. The students had not demonstrated success in multiplication with regrouping in other instructional settings. The CRA-SIM intervention was short, but explicit and focused. This has implications for its use as a tertiary intervention. For the students in this study, their performance increased to levels expected for students at their grade level after 80 minutes of instruction per week (i.e., 20 minutes, four days per week) across three weeks for a total of 240 minutes. In addition, the students' demonstration of conceptual understanding will provide for cognitive structures that students can use when presented with more complex operations within the standards in the coming grade level. The amount of time invested would not preclude the provision of other interventions and the daily time investment is feasible within a school day. The materials needed were not expensive or outside of typical resources used within elementary mathematics such as base ten blocks.

Another significant component of this intervention is its inclusion of mathematical practices in addition to computation and procedural instruction; all three are important components of mathematics understanding and proficiency. The students demonstrated understanding and articulated their conceptualization of multiplication as an operation and its relation to addition and subtraction. The students represented the operation using objects and drawings as well as demonstrated proficiency and precision in computation.

Teachers may use the results of this study along with other related literature to emphasize modeling and representation within computation interventions. Students who have mathematics difficulties may have a weak understanding of numbers. Without this understanding, computation may become a set of procedures to be memorized. Physical representation, modeling, and description used within CRA bring meaning to the computation process. In addition, teachers may use the results from this study to incorporate more opportunities for application within computation interventions. The inclusion of application and word problems did not interfere with students' progress in computation and students maintained their use of computation within the context of word problems after instruction.

Additional research is needed to address limitations previously discussed related to implementation by classroom teachers and comparison to other explicit interventions. However, this study extends research and provides additional evidence that may lead to further support of CRA-SIM as an effective practice for teaching multiplication with regrouping in alignment with mathematics standards.

References

Common Core State Standards Initiative (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Flores, M. M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure, 53*(3), 145–152.

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education, 31*(3), 195–207. https://doi.org/10.1177%2F0741932508327467

Flores, M. M., & Franklin, T. M. (2014). Teaching multiplication with regrouping using the concrete-representational-abstract sequence and the Strategic Instruction Model. *Journal of the American Academy of Special Education Professionals*, *6*(3), 133–148. http://aasep.org/aasep-publications/journal-of-the-american-academy-of-special-education-professionals-jaasep/jaasep-fall-2014/teaching-multiplication-with-regrouping-using-the-concrete-representational-abstract-sequence-and-the-strategic-instruction-model/index.html

Flores, M. M., Hinton, V. M. & Strozier, S. D. (2014). Teaching subtraction and multiplication with regrouping using the concrete-representational-abstract sequence and strategic instruction model. *Learning Disabilities Research & Practice, 29*, 75–88. doi:10.1111/ldrp.12032

Flores, M. M., Schweck, K. B., & Hinton, V. M. (2014). Teaching multiplication with regrouping to students with specific learning disabilities. *Learning Disabilities Research & Practice, 29*, 171–183. doi:10.1111/ldrp.12043

Horner, R. H., Carr, E. G., Halle, J., McGee, G., Odom, S., & Wolery, M. (2005). The use of single-subject research to identify evidence-based practice in special education. *Exceptional Children, 71*, 165–179.

Hosp, M. K., Hosp, J. L., & Howell, K. W. (2007). *The ABCs of CBM: A practical guide to curriculum-based measurement.* New York: Guilford.

Hudson, P. P., & Miller, S. P. (2006). *Designing and implementing mathematics instruction for students with diverse learning needs.* Boston MA: Pearson.

Lembke, E. S., Hampton, D., & Beyers, S. J. (2012). Response to intervention in mathematics: Critical elements. *Psychology in the Schools, 49.* 257–272. doi:10.1002/pits.21596

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *15,* 10–21.

Maccini, P., & Ruhl, K. L. (2000). Effects of graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23,* 465–489.

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research and Practice, 27,* 152–166. doi:10.1111/j.1540–5826.2012.00363.x

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*, 19–35.

Miller, S. P., & Kaffar, B. J. (2011). Developing addition with regrouping competence among second graders with mathematics difficulties. *Investigations in Mathematics Learning, 4*(1), 24–49.

Miller, S. P., Stringfellow, J. L., Kaffar, B. J., Ferreira, D., & Mancl, D. B. (2011). Developing computation competence among students who struggle with mathematics. *Teaching Exceptional Children*, *44*(2), 38–48. http://cec.metapress.com.spot.lib.auburn.edu/content/n7515435771053qp/?p=a51dc4e867f74b32a1788b53f6075ca2&pi=3

Morin, V. A., & Miller, S. P. (1998). Teaching multiplication to middle school students with mental retardation. *Treatment of Children*, *21*(1), 22–36.

Parker, R. I., Vannest, K. J., Davis, J. L., & Sauber, S. B. (2011). Combining nonoverlap and trend for single-case research: Tau-U. *Behavior Therapy, 42,* 284–299. doi:10.1016/j.beth.2010.08.006

Peterson, S. K., Mercer, C.D., & O'Shea, L. (1988). Teaching learning disabled children place value using the concrete to abstract sequence. *Learning Disabilities Research, 4,* 52–56.

Poling, A., Methot, L. L., & LaSage, M. G. (1995). *Fundamentals of behavior analytic research.* New York: Plenum.

Scruggs, T. E., & Mastropieri, M. A. (2013). PND at 25: Past, present, and future trends in summarizing single-subject research. *Remedial and Special Education, 34*, 9–19. doi:10.1177/0741932512440730