

Using the Strategic Math Series and Evidence-Based Practices to Improve Achievement

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Mathematics is challenging for many students with disabilities and students receiving tiered interventions, and their achievement is a concern among teachers. Difficulties in mathematics are often related to deficits in conceptual and procedural knowledge of numbers and operations and a lack of problem-solving skills (Miller et al., 2011; Powell et al., 2023). The increased emphasis on standards-based instruction requires teachers to use effective and efficient practices. Mathematics standards prioritize the use of varied approaches to computation that requires conceptual understanding of numbers, operations, and relations between operations. Therefore, evidence-based practices must be implemented for students to demonstrate conceptual and procedural knowledge and problem-solving skills in mathematics. Using the *Strategic Math Series* and evidence-based practices improves achievement (Flores et al., 2016; Flores et al., 2019; Miller & Kaffar, 2011; Milton et al., 2019).

Evidence-Based Practices

The literature on mathematics interventions reveals improved achievement when teachers use (a) the concrete-representational-abstract (CRA) teaching sequence (Flores et al., 2016; Milton et al., 2019); (b) explicit teaching principles (Kaffar, 2014; Mancl et al., 2012); (c) cognitive strategies (Flores et al., 2020; Miller & Kaffar, 2011); and (d) a graduated sequence of lessons (Flores et al., 2019; Kaffar, 2014; Miller & Kaffar, 2011; Milton et al., 2019) to teach computation and problem-solving skills. When the *Strategic Math Series* and evidence-based practices are implemented, students with disabilities and students receiving tiered interventions improve mathematics achievement.

Concrete-Representational-Abstract Teaching Sequence

The concrete-representational-abstract (CRA) teaching sequence promotes conceptual and procedural knowledge related to addition, subtraction, multiplication, and division (Miller et al., 2011; Powell et al., 2023). The CRA teaching sequence begins with the use of manipulative objects (i.e., concrete), transitions to the use of pictures or tallies (i.e., representational), and progresses to solving problems using numbers only (i.e., abstract). Conceptual knowledge in

mathematics is an understanding of the relationships and connections among operations. Algorithms are the step-by-step procedures for solving problems. Procedural knowledge in mathematics is the ability to use a step-by-step process to solve problems with accuracy. Each phase builds on the previous phase of the CRA teaching sequence to promote conceptual and procedural knowledge and mathematics fluency (Flores et al., 2019; Miller et al. 2011; Powell et al., 2023). Conceptual and procedural knowledge are essential for improving mathematics achievement for students with disabilities (Flores et al., 2016; Kaffar, 2014; Miller & Kaffar, 2011).

Concrete phase. During the concrete phase of instruction, students use three dimensional objects (e.g., interlocking cubes, plastic chips, or base-ten blocks) to represent and solve problems. Base-ten blocks are manipulative objects (e.g., hundreds flats, tens strips, and ones blocks) for solving problems involving place value and regrouping. Place value in the base-ten system uses ten digits (0 to 9) to represent a number of items with groups of ones, tens, hundreds, and so on. Regrouping in mathematics is the process of trading, or renaming, a group of ten ones, ten, hundred, and so forth within the base-ten system.

Representational phase. During the representational phase of instruction, students use pictures or drawings (e.g., squares, lines, or tallies) to solve problems. When the computation involves place value (i.e., hundreds, tens, and ones) and regrouping, pictures of squares represent hundreds, vertical lines represent tens, and tallies represent ones for multi-digit computation. When providing instruction with pictures or drawings, it is important to also include the numerical problem to develop conceptual and procedural knowledge of the computational skills.

Abstract phase. During the abstract phase of instruction, students use numbers only to solve problems without objects or drawings for computation. Often, students transition from the representational phase to the abstract phase by applying cognitive strategies (e.g., mnemonic devices) to prompt recall of the algorithm to solve problems (Flores et al., 2019; Kaffar, 2014; Miller et al., 2011). See Figure 1 for examples of mnemonic devices for multiplication with regrouping. Also, the abstract phase focuses on fluency (i.e., automaticity) of computation; fluency in mathematics is the ability to memorize factual information and recall it with automaticity.

Explicit Teaching Principles

Explicit instruction involves clear, accurate, and supportive teaching procedures. Research supports the use of explicit teaching principles for teaching mathematics to students with disabilities and students receiving tiered interventions (Flores et al., 2020; Mancl et al., 2012; Milton et al., 2019; Powell et al., 2023). The instructional procedures for mathematics lessons include the following: (a) advance organizer, (b) describe and model, (c) guided practice, (d) independent practice, and (e) feedback. Error patterns in mathematics are an inability to solve problems with accuracy and may involve a lack of conceptual knowledge or procedural knowledge. Proper feedback after each lesson will reduce error patterns and improve achievement. Additionally, lessons should include problem-solving practice (i.e., word problems) with explicit guidance and support to promote the relevance and importance of learning the skills (Flores et al., 2019; Kaffar, 2014). This will assist students with making the connection between computation and word problems.

Cognitive Strategies

Cognitive strategies (e.g., mnemonic devices) are used in mathematics to help students remember the step-by-step process (i.e., algorithm) for computation and problem-solving. The RENAME Strategies (Flores & Kaffar, 2018a; Flores & Kaffar, 2018b) are examples of first-letter mnemonic devices for multiplication with regrouping (see Figure 1). It is important for students to practice and memorize the mnemonic device. The CRA teaching sequence and cognitive strategies have been used simultaneously to support mathematics instruction (Flores et al., 2016; Mancl et al., 2012; Miller et al., 2011; Miller & Kaffar, 2011).

Graduated Sequence of Lessons

A graduated sequence of lessons involves the implementation of efficient and systematic methods to facilitate mathematics achievement (Kaffar, 2014). After a pretest to assess prerequisite skills, lessons are carefully sequenced and build upon each other in terms of complexity. There is an emphasis on the development of conceptual and procedural knowledge and fluency. Additionally, the CRA teaching sequence is implemented in conjunction with explicit teaching principles and cognitive strategies during the graduated sequence of lessons. For example, to teach multiplication with regrouping, instruction begins with two-digit computation without regrouping in the first lesson and becomes more complicated with each lesson until two-digit computation with regrouping up to four times is required. Each lesson includes problem-solving practice (i.e., word problems), and the word problems gradually become more complex to match the sequence of lessons. Each phase of learning (i.e., concrete, representational, and

abstract) must be taught to mastery to ensure that students acquire and retain the skills; thus, consistent assessment is important for students to transition from one phase to the next in an efficient manner (Flores et al., 2016; Miller et al., 2011). Furthermore, mastery of conceptual and procedural knowledge is essential to improve mathematics achievement for students with disabilities (Powell et al., 2023). Administering a posttest will show whether the skills are mastered, and supplemental practice is recommended for maintenance of the skills and to improve fluency (i.e., automaticity). The graduated sequence of lessons is an effective and efficient method to improve conceptual and procedural knowledge related to computation and problem-solving.

Summary

Students with difficulties in mathematics need instruction that addresses their deficits in conceptual and procedural knowledge. This instruction ensures that students understand the relationships and connections among operations and can follow step-by-step procedures for computation and problem-solving. When the *Strategic Math Series* and evidence-based practices, such as the CRA teaching sequence, explicit teaching principles, and cognitive strategies are implemented along with a graduated sequence of lessons, students with disabilities and students receiving tiered interventions improve mathematics achievement.

Figure 1. *RENAME Strategies for Partial Products and Standard Algorithm*

RENAME Strategy for Partial Products	RENAME Strategy for Standard Algorithm
Step 1: Read the problem.	Step 1: Read the problem.
Step 2: Examine the ones column to make equations.	Step 2: Examine the ones column: 10 or more, go next door.
Step 3: Note the partial products for ones.	Step 3: Note the ones.
Step 4: Address the tens column to make equations.	Step 4: Address the tens column: 10 or more, go next door.
Step 5: Mark the partial products for tens.	Step 5: Mark the tens.
Step 6: Examine the columns, add, and check.	Step 6: Examine the columns, begin again, or add and check.
<p>“10 or More” Sentence</p> <p>If multiplying the numbers results in 10 or more, regroup the tens and add to the total in the next column (10 or more, go next door).</p>	

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