

## Strategic Math Series: Part Two

### —Programming for Effective Teaching of Basic Math Facts —

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**"Unfortunately, many students with mild disabilities fail to achieve an understanding of basic math facts or develop fluency in using facts."**

**M**ath has a logical structure. Students construct simple relationships first and then progress to more complex tasks. As the student progresses in this ordering of math tasks, the learning skills and content transfer from each step to the next. In their *Twelve Components of Essential Mathematics*, the National Council of Supervisors of Mathematics highlights the need for students to be knowledgeable of basic facts and proficient in basic operations (addition, subtraction, multiplication, division). Many authorities claim that failure to understand basic concepts in beginning math instruction contributes heavily to later learning problems. Unfortunately, many students with mild disabilities fail to achieve an understanding of basic math facts or develop fluency in using facts. Given the problems that students with learning disabilities exhibit with lower level skills (for example, many students do not know the 390 basic math facts after five or more years of school) and the importance of these skills to overall math achievement, it is apparent that comprehensive programming to teach basic skills is needed.

Educators who have examined the mathematical deficits of students have suggested a number of initial teaching and remediation methods. Many of these methods feature the concrete-representational-abstract (CRA) teaching sequence (for example, use of objects, use of pictures and tallies, use of numbers only) that has been found to facilitate math learning. Implicit in this method of instruction is an emphasis on teaching students to understand the concepts of mathematics

prior to memorizing facts, algorithms, and operations. Although the CRA sequence is widely advocated for mathematical learning, it is rarely used in a systematic manner during math instruction. The *Strategic Math Series* for facts provides a systematic means of CRA instruction.

According to the CRA sequence, instruction begins at the **concrete level** where the student uses three dimensional objects to solve computation problems. For example, in solving the problem, 5 minus 2, the student is instructed to look at the second number, 2, and remove the equivalent number of objects from the group of 5 objects. After being instructed to count the remaining objects, 3, and write the answer, the student says the answer to the problem. After successfully solving numerous subtraction problems at the concrete level, the student proceeds to the **representational level**.

At the **representational level**, drawings are used to solve computation problems. For example, in solving the problem 7 minus 3, the student is instructed to look at the first number, 7, and draw that number of tallies. Next the student is instructed to look at the second number, 3, and eliminate the same number of tallies, 3, to arrive at the answer. Finally, the student says and writes the answer to the problem. After successfully solving several subtraction problems at this level, the student begins to work at the next level, the **abstract level**.

At the **abstract level**, the student looks at the computation problem and tries to solve it

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without using objects or drawing. The student reads the problem, remembers the answer or thinks of a way to compute the answer, and writes the answer. No objects or drawings are used in the computation unless the student is unable to answer a problem. Since success in math requires the ability to solve problems at the abstract level, it is essential that students achieve mastery at this level.

Using the CRA sequence, the Strategic Math Series contains two levels of instruction. Level 1 is designed primarily for elementary students, and Level 2 is appropriate for secondary students and beyond. Both levels of instruction contain research-tested teaching procedures that enable all students with math difficulties to understand, remember, and apply important mathematical skills.

### Level One

The first level of instruction, of which this manual is a part, is comprised of seven manuals. Although Level 1 manuals were initially developed for use with elementary students with learning problems, they can also be used with secondary students who need remedial instruction. Each manual at this level is designed to teach one of the following mathematical skills: addition facts 0 to 9, subtraction facts 0 to 9, addition facts 10 to 18, subtraction facts 10 to 18, multiplication facts to 81, division facts to 81, and place value. Manuals may be used either individually (for example, in a resource room to help students with a particular skill), or collectively for teaching facts in a self-contained classroom or a remedial math program. Although each manual is self-contained, some of the manuals assume a certain degree of prerequisite knowledge on the part of the student. For example, for *Subtraction Facts 0 to 9*, the

student should master the skills taught in *Addition Facts 0 to 9*. For these reasons, the following sequence is recommended:

1. Addition Facts 0 to 9
2. Subtraction Facts 0 to 9
3. Place Value
4. Addition Facts 10 to 18
5. Subtraction Facts 10 to 18
6. Multiplication Facts 0 to 81
7. Division Facts 0 to 81

As presented in Table 1, the instructional sequence of the Strategic Math Series is divided into seven phases with twenty-one basic lessons. Student completion of all twenty-one lessons is important for two reasons. First, the lessons are sequenced and build upon each other in terms of complexity. Second, although most students acquire the respective computation skill (e.g., multiplication facts) when they reach Posttest, they need additional practice to maintain their knowledge and skills, to increase their fluency, and to ensure further development of their problem-solving skills.

### Instructional Phases of the Strategic Math Series

*Phase 1: Pretest*. During this

instructional phase, a *Pretest* is administered to the student to determine whether instruction is needed. If a student's score on the *Pretest* falls below the mastery criterion (i.e., 80%), he is informed that he needs to work on the targeted basic facts. The need for instruction is discussed, and a commitment to learn is obtained from the student via a signed contract.

*Phase 2: Teach concrete application*. The concrete phase of instruction includes Lessons 1-3. For each lesson, a sample script and learning sheets guide the teacher through the instructional sequence. During these lessons, students manipulate concrete objects to solve basic facts on their learning sheets. Students also begin to solve word problems in which the numbers are vertically aligned, but blank spaces are provided after the numbers for students to write the name of the manipulative object, and then the students read the problem. These concrete lessons act as a springboard for learning facts at the representational and abstract levels.

*Phase 3: Teach representational application*. The representational phase of instruction includes Lessons 4-6. Again, a sample script and learning sheets guide the teacher

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**Table 1**  
**Instructional Phases of the Strategic Math Series**

<i>Phase</i>	<i>Purpose</i>	<i>Lessons</i>
1	Pretest	Pretest Lesson
2	Teach Concrete Application	Lessons 1-3
3	Teach Representational Application	Lessons 4-6
4	Introduce the "DRAW" Strategy	Lesson 7
5	Teach Abstract Application	Lessons 8-10
6	Posttest	Posttest Lesson
7	Provide Practice to Fluency	Lessons 11-21

(continued from page 2)

through each lesson. In this phase, students use drawings and tallies to solve basic facts on their learning sheets. Moreover, they continue to solve word problems in which the numbers are vertically aligned, but now they fill in the blanks after the numbers with the name of the drawing, and then the students read the problem. Representational lessons help students understand the respective facts as they move toward the abstract level.

**Phase 4: Introduce the "DRAW" strategy.** Many students with learning difficulties are passive when faced with a problem-solving situation (i.e., they tend to guess or quit working). However, these same students can become active and independent learners when they master a problem-solving strategy to facilitate computation. Thus, Lesson 7 introduces a math strategy called "DRAW" to help students solve facts at the abstract level. Each letter of "DRAW" cues students to perform certain procedures:

1. Discover the sign
2. Read the problem
3. Answer, or "DRAW" and check
4. Write the answer

**Phase 5: Teach abstract application.** This phase of instruction is presented in Lessons 8-10. For

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each lesson, a script guides the teacher through the instructional sequence. Again, a learning sheet is provided to facilitate continued student practice of the targeted facts. During this time, students use the "DRAW" Strategy to solve abstract-level problems when they are unable

to recall an answer. Students also begin to solve word problems in which the numbers are still vertically aligned but now include the names of common objects or phrases after the numbers instead of blank spaces.

**Phase 6: Posttest.** During this phase of instruction, a Posttest is administered to each student to determine whether students have learned the basic facts and are ready to proceed to the phase of instruction

**"Each lesson features a script to guide the teacher through the instructional sequence, plus a learning sheet to facilitate student practice of facts and word problems."**

designed to increase fluency (speed of computation) and further develop problem-solving skills.

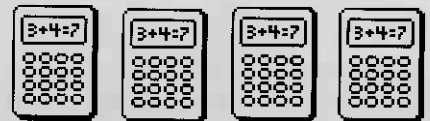
**Phase 7: Provide practice to fluency.** The practice to fluency phase takes place in Lessons 11-21. Each lesson features a script to guide the teacher through the instructional sequence, plus a learning sheet to facilitate student practice of facts and word problems. Students work on three primary skills: (a) solving word problems that become increasingly complicated as the lessons progress; (b) increasing the rate at which they can compute facts; and (c) discriminating previously learned facts from the newly acquired facts and accurately computing those problems.

To help students solve more complicated word problems, specific practice is provided in Lessons 11-21. Problems are presented in sentence form (as opposed to the numbers being vertically aligned with phrases written to the right of the numbers). As the lessons progress, students learn to filter out extraneous information and to create their own word problems.

To help students increase their rate of computation, a one minute timed probe called Addition, Subtraction, Multiplication, or Division Minute is given during selected lessons of this phase. A student is considered to be fluent or to have reached mastery on the Minute probes when he is able to write the answers to problems at the rate of 30 digits per minute with no more than two errors.

Finally, to help students discriminate between types of facts, all students receive a one page *Facts Review* containing two or more types of facts during selected lessons of this phase. Such practice not only checks the students' ability to discriminate facts when presented on the same page, but also provides important practice of previously learned facts.

The instructional sequence of the *Strategic Math Series* includes many of the components of effective math instruction highlighted in the previous issue of *Strategram*. The instructional procedure, the relationship of the *Strategic Math Series* design to the components of effective math instruction, and field test results will be reported in the next *Strategram*.



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## **Where Do You Fit in the "BIG" Picture (of SIM Implementation)?**

By  
Frank Kline

Have you attended all the workshops you can, learned all the strategies you can and still feel that you need to grow in your implementation? Have you ever wondered what else you could do with the strategies? Have you ever wished for an opportunity to have a customized inservice program individualized around your particular needs and abilities? Have you ever wondered what an ideal strategies program would look like? Have you ever wished for a "bird's eye view" of someone else's strategies program? Perhaps what is missing is a "big picture" of the Strategies Instruction Model. An innovation configuration is one way to develop such a big picture.

Practitioners and innovators have known for some time that any innovation or new program such as the Strategies intervention Model is implemented differently from one setting to another. Minor variations can have either a minimal or a positive effect on the outcome of the program. These variations result naturally because different settings

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**"If a very poorly implemented innovation is rejected, one cannot be sure whether the rejection is due to the quality of the implementation or the quality of the innovation."**

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create unique set of constraints. An example of a minor variation might be an altered scoring sheet in order to accommodate larger classes. At the other end of the spectrum, major changes leave the innovation changed so markedly that it is difficult to recognize. A major change such as ignoring generalization because it is difficult to enlist the cooperation of a content area teacher would substantially affect the outcomes of strategy implementation. Particularly in the early stages of adoption, it is critical to have a relatively "pure" implementation of an innovation. If a very poorly implemented innovation is rejected, one cannot be sure whether the rejection is due to the quality of the implementation or the quality of the innovation.

The variability of new program implementation has led to the development of the Strategies Intervention Model Configuration Checklist for the

Strategies Intervention Model (SIM). The Checklist, shown on pages 7 and 8, is designed to show the first two parts of the critical variables of the SIM and the range of variation. The University of Kansas Institute for Research in Learning Disabilities worked to develop this Checklist for the Strategies Intervention Model during the 1990-1991 school year. The Checklist was based on: (1) informal observations by the KU-IRLD staff of both model and unacceptable types of implementation; (2) formal observations and interviews of fourteen teachers implementing more than one of the learning strategies; and (3) comments on a working draft of the Checklist by a nationwide sample of seventy-five teachers using the Strategies Intervention Model.

### **Description of the Strategies Intervention Model Configuration Checklist**

The Strategies Intervention Model Configuration Checklist for the SIM is divided into three parts: curriculum, instruction, and environment. On pages 7 and 8 are the critical elements for the curriculum and instruction sections. These parts have several critical elements, each with two or three descriptors; the ideal implementation is described at the left side of the page. There are a total of thirty-one critical elements represented across all three parts. Part One of this series describes the curriculum and instruction sections. Selected critical elements will be described individually. In addition, this article will focus on how a teacher can place their program on the Strategies Intervention Model Configuration Checklist for the SIM. Part II of this series (Strategram, Vol. 4, No. 3) will focus on the environment section of the checklist and on how the checklist can be used to create an individualized professional development plan.

#### Curriculum

The first five critical elements deal with the curricular aspects of SIM implementation. Specifically included are the Learning Strategies Curriculum, the Social Skills Curriculum, The Educational Planning Strategy, the motivational aspects of the strategies, and non-KU-IRLD strategies. This section is designed to answer the question, "What strategies are being taught in your particular implementation of SIM?" These elements of the SIM, those most often covered in training sessions, are the most visible elements.

Two specific critical elements should be pointed out. First, item number 4 deals with motivational

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elements. The item specifically mentions motivational elements contained in the manuals and motivational elements beyond the manual. Motivation is being increasingly recognized as a key component in strategy instruction. Several specific motivational devices are built into strategy instruction; obtaining student commitments, the use of rationales,



goal setting, presenting expected results, forecasting generalization activities, and charting student progress can all aid in motivating the reluctant student. These and other motivational elements are built into the scripts for the strategies. In the end, however, motivation is personal. All of us have felt at some time or another the inability to motivate a student. That is the reason that elements outside of the manual are also mentioned in item number 4. The manual is replete with motivational techniques. If they are not sufficient for a specific student, a teacher should not hesitate to include motivational techniques that have proven effective with the student in the past or to try new methods.

Second, item 5 addresses non-KU-IRLD strategies. There are a broad range of cognitive strategies have been successfully used, although only a few have been made available in teacher friendly packages. However, the strategy teacher with extensive experience in the SIM could use the stages of acquisition and generalization to provide instruction in these strategies. This allows the teacher to become comfortable in their use and aware of their potential application.

### Instruction

The Instruction section of the SIM Configuration Checklist for the SIM starts with item number 6 and runs through item number 18. The focus of this section is on how the strategies are taught. Specifically covered in these critical elements are the use of the stages of acquisition and generalization in strategic and non-strategic content, the use of content enhancement devices, and the use of appropriate feedback. Also included are general

independence building behaviors such as encouraging student self talk, enabling student problem solving in response to specific setting demands, and the appropriate provision of directions. This section answers the question, "How are strategies being taught?"

Three critical elements in the instruction section may need some additional explanation. Item number 10 refers to independent self-regulation, independence, and direction of students. The goal of the SIM is, of course, independent self-regulation. However, most students are not performing at that level. The teacher's challenge is to determine what level of support the student needs and provide it while attempting to move the student toward independence. A student working at the independent self-regulation level needs a structure that allows him/her to function freely with minimal interference from the teacher. The student working at the level of independence needs occasional prompts to cue him/her to focus while other students may require specific direction or need more careful monitoring of his/her work habits.

Item number 16 refers to a system of program evaluation. Each strategy includes certain elements of program evaluation in the regular charting of students' progress. The full implementation of SIM requires a further level of program evaluation focused not on the individual strategy but on the program as a whole. Evaluation tools used for this purpose include not only the specific strategy records but general measures of attitude and achievement valued by the local community. Student grades would probably be a significant part of this evaluation. In addition, other measures of attitude and achievement (formal or informal) that seem sensitive to strategy instruction and are valued by the local community schools may be employed. Collection of this type of data is

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important because it can be used to show student progress to you and your students as well as to others (i.e., teachers, administrators, parents, other students, etc.). Unless the data is collected, formatted in an easily understood fashion, and communicated to others, the accomplishments of your program will not be understood.

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Finally, item number 18 represents encouragement of student independence in dealing with pertinent setting demands. There are three parts to this critical element. Enabling executive functioning refers to minimal cues that allow a student to recall a method of dealing with a similar problem or an opportunity to use a specific strategy. Instruction in problem solving means telling the student how to figure out the problem. It does not mean solving the problem. It does mean focusing on the method for problem solution. Modeling problem solving/executive functioning means showing the student how to self talk through a plan of action for the specific problem under consideration. This last method is only slightly more than telling the student what to do. Ideally, this support should be provided only to the extent needed, such as the example for item number 10.

### Placing Your Program on the SIM Configuration Checklist

The SIM Configuration Checklist is used by evaluating which descriptor related to each critical element characterizes a program and placing a check in the appropriate column. This provides a profile of a particular program that is useful in many ways. The use of the profile to make your own professional development plan will be covered in Part II of this article. The teacher can develop a profile of his/her program by completing a self assessment of the program.

#### SIM Self Assessment

#### Self Assessment

There are certain methods that may assist a teacher in making an accurate self assessment. The first requirement is a desire for honest self appraisal. Second, the teacher should read carefully each of the critical items and form an initial impression on where his or her program fits. Next, the teacher should consider if a document exists which will help confirm his/her first impression. Documents that may be considered would include: IEP's, student progress charts, lesson plans, form letters sent to parents, statements of philosophy, district/building policies, etc. Third, if no documents exist that can assist in placement on this critical element, the teacher should try to think of specific incidents that illustrate the

impression of the placement of his/her program. The more specifically an element can be recalled, the more certain the teacher can be of an accurate placement. It is at this point that clarity of memory and honesty of desire play a critical role. After deciding which of the descriptions best describe his/her program, the teacher should place a check in that column. This process will result in a profile of a program that shows how it is related to an "ideal" program as conceived by the developers and confirmed by many teachers and trainers using the SIM.

**HAPPY HOLIDAYS TO YOU AND  
YOUR FAMILY FROM EVERYONE  
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## STRATEGIES INTERVENTION MODEL Configuration Checklist

Curriculum				
Ideal Implementation	√		√	
1. Teaches most learning strategies they know, at least in all three strands.		Teaches about 50 % of the learning strategies they know in at least two strands.		Teaches more than one learning strategy but in only one strand.
2. Teaches social skills as necessary (i.e, body basics to all students and other skills as students need them).		Teaches social skills to all students regardless of individual need.		
3. Includes <u>The Educational Planning Strategy</u> in program. Cues use of the strategy and refers to student long-term goals frequently.		Teaches <u>The Educational Planning Strategy</u> . Cues use only for IEP conference.		Teaches <u>The Educational Planning Strategy</u> but does not cue use anywhere.
4. Implements the motivational aspects in manuals as well as other motivational techniques in all instruction.		Implements the motivational aspects imbedded in manuals as well as other motivational techniques for strategy instruction only.		Uses all motivational aspects imbedded in manuals as the strategies are taught.
5. Teaches non-IRLD strategies when appropriate.		Overtly models use of non-IRLD strategies.		
Instruction				
6. Implements all stages of acquisition as written in the manual and goes beyond the manual as necessary for a particular student.		Implements all stages of acquisition as written in the manual.		Implements all stages of acquisition but leaves out parts (i.e., does no goal setting in describe stage).
7. Regularly moves most of the students into generalization (about 100%).		Moves most of the students into generalization (about 80%).		Moves few students into generalization (about 40%).
8. Has adapted strategy instruction to reflect the new phases of generalization.		Implements all phases of generalization as written in the manual and refers to generalization in previous stages of acquisition.		Implements all phases of generalization as written in the manual.
9. Makes regular reference to and prompts and/or models the use of IRLD strategies that might be helpful during non-strategy instruction.		Uses the IRLD strategies as non-strategy content is taught identifying them as they are used.		Uses the IRLD strategies as non-strategy content is taught without reference to them.
10. Uses language and behavior that promotes independent self regulation, prompts independence, or directs students in an appropriate manner.		Uses language and behavior that prompts independence or directs students in an appropriate manner.		

**STRATEGIES INTERVENTION MODEL  
Configuration Checklist (cont.)**

Instruction					
Ideal Implementation	√		√		√
11. Encourages student's self talk throughout strategy instruction.		Models self talk as called for in manual.			
12. Personalizes the instruction to the students using extra rationales and examples related directly to the students and timed for maximum effect.		Includes some personal aspects in the instruction but only as the manual calls for them.		Includes only the rationales and examples included in the manual with no effort to personalize them.	
13. Plans for and incorporates content enhancement devices including those in the manuals into instruction on a regular basis.		Plans for and incorporates content enhancement devices including those in the manuals into instruction of either strategies or non-strategy content on a regular basis.		Plans for and uses the content enhancement devices in the manuals only for strategy instruction.	
14. Evaluates student products promptly and accurately, and ensures appropriate record keeping by the student.		Evaluates student products promptly and accurately, and keeps appropriate records.		Evaluates student products accurately and promptly.	
15. Provides knowledge of results to all and elaborated feedback to the students who need it the most.		Provides knowledge of results and elaborated feedback to all students.		Provides knowledge of results but no elaborated feedback.	
16. Has an ongoing program evaluation process illustrating effects of strategy instruction using a variety of sources. Uses this information with the proper consumers.		Has an ongoing program evaluation process illustrating effects of strategy instruction using a variety of sources. Uses this information with the students only.		Keeps records of acquisition and generalization steps completed. Uses with students only.	
17. Uses SIM stages of acquisition and generalization appropriately to teach non-strategy content as well as strategies.		Uses SIM stages of acquisition and generalization appropriately to teach strategies.			
18. Enables student problem solving/executive functioning in response to specific setting demands.		Instructs students in problem solving/executive functioning in response to specific demands.		Models problem solving/executive functioning in response to specific setting demands.	

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