# Introduction

### **Contents of Manual**

This manual contains the materials needed to teach the standard algorithm for multiplication with regrouping using the Concrete-Representational-Abstract (C-R-A) method of instruction with an emphasis on the mathematical practices infused throughout the Numbers and Operations standards in most states. The materials allow for computation instruction within the context of meaningful problem situations. As students master and demonstrate understanding of multiplication with regrouping, the materials assist them in understanding its relation to other operations.

### **Students Who Need This Manual**

Students who consistently solve problems as shown below may need *Multiplication With Regrouping*.

2	3				
2 4	2	2 5	3 4	2 4	3 3
x 3 6	5 <u>x 1</u>	7	x 2 3	x 6	x 6
8 4	7	' 5	6 1 2	224	1,818

The errors above are examples of those made by students who later benefited from this program. The errors show that students lack a sense of numbers, that multi-digit numbers are not just separate numerals, but each one has a different value (47 is 4 tens and 2 ones rather than a 4 and 2). Students who have attempted to memorize steps to the algorithm without a sense of numbers engage in various types of error patterns such as those above. The purpose of this program is to build students' sense of numbers and understanding of the multiplication operation. In addition, the program is about understanding the operation in the context of real-life situations. So, each lesson presents computation problems with words that build into word problems and finally progress to differentiation between addition, subtraction, and multiplication problems. This allows students to engage in mathematical practices.

#### **Results of the Field Test**

*Multiplication With Regrouping* lessons were field tested with elementary and middle school students with disabilities as well as with students at-risk for failure who received the intervention within a tiered prevention model. Three studies were conducted using single case designs (Flores, Hinton, & Strozier, 2014; Flores, Schweck, Hinton, 201; Flores & Hinton, in press). Two additional studies involved larger groups of students and the results were analyzed using statistical methods; one study compared *Multiplication With Regrouping* with a Direct Instruction intervention program (Flores & Franklin, 2014; Flores & Kaffar, under review). All field testing (five studies to date) occurred in public schools and *Multiplication With Regrouping* lessons were used to supplement the students' regularly scheduled mathematics instruction.

Flores, Hinton, and Strozier (2014) conducted the first study in which three third-grade students receiving tertiary (tier three) intervention participated in instruction during an

intervention class period within the school day. A multiple baseline across behaviors design demonstrated a functional relation between *Multiplication With Regrouping* lessons and fluency in computing multiplication with regrouping. The students' fluency in solving equations increased from zero correct digits to at least 30 correct digits after participating in instruction.

Flores, Schweck, and Hinton (2014) conducted the second study in which four fifth grade students with specific learning disabilities received instruction during a portion of their daily mathematics instruction in a special education resource room. A multiple probe across-students design demonstrated a functional relation between *Multiplication With Regrouping* lessons and student performance, increasing fluency from zero correct digits to 30 correct digits. In addition, all of the students maintained their learning after instruction ended and transferred their learning to a near-generalization task.

Flores and Franklin (2014) conducted a study in which six fourth-grade students receiving tertiary (tier three) interventions participated in the study. A general education teacher taught the strategy within an after-school intervention program. The researchers measured students' accuracy in solving multiplication with regrouping problems before and after instruction. The researchers observed 40% of the lessons and completed a checklist of instructional behaviors. The teacher implemented the strategy with 93% accuracy across lessons. The teacher reported that she would use the strategy again. An analysis of variance statistical procedure showed that students made significant progress in computation.

Flores and Hinton (in press) taught three third-grade students receiving tertiary (tier three) intervention. A multiple baseline across-students design demonstrated a functional relation between *Multiplication With Regrouping* lessons and fluency in computing multiplication with regrouping. The students' fluency in solving equations increased from zero correct digits to at least thirty correct digits.

Flores and Kaffar (under review) conducted a study in which fifteen matched pairs of students with and without disabilities in Grades 3 through 6 were randomly assigned to either *Multiplication With Regrouping* or a Direct Instruction multiplication intervention program. Special education teachers implemented the program. The researchers observed 53% of the lessons and completed treatment integrity checklists. The teachers implemented the program with 93% accuracy. The teachers reported that they liked the program and would recommend it to others. Both groups made statistically significant progress with regard to computation accuracy, but students who received *Multiplication With Regrouping* outperformed their peers with regard to fluency.

#### **Prerequisite Skills**

In order to benefit from instruction using *Multiplication With Regrouping*, students should have mastered basic multiplication facts (zero through five) plus addition and subtraction involving regrouping. If students have not mastered these skills, instruction should be provided prior to taking the *Multiplication With Regrouping* Pretest. Field-testing has shown that deficits in these areas will significantly interfere with student learning. The problems in this manual were intentionally developed so that students who have not mastered facts involving larger numbers such as 8x7 or 6x9 can be successful. Manuals within the Strategic Math Series can be used to

remediate deficits in the aforementioned areas. These include *Addition Facts 0 to 9* (Miller & Mercer, 1991a), *Subtraction Facts 0 to 9* (Miller & Mercer, 1991b), *Multiplication Facts 0 to 81* (Miller & Mercer, 1993), *Addition With Regrouping* (Miller, Kaffar, & Mercer, 2011), and *Subtraction With Regrouping* (Miller, Kaffar, & Mercer, 2011).

## **The Instructional Sequence**

This manual is divided into eight instructional phases that include 18 lessons in which students compute problems and solve word problems. Instruction in word problems involves discrimination between operations to ensure understanding of multiplication. Lessons build upon each other, increasing in complexity as students make progress through the program. If students miss a lesson or perform poorly (less than mastery – see table below), it is important that they receive instruction and remediation rather than moving ahead to keep pace.

Phase	Purpose	Lessons	Mastery Criteria
Phase 1	Pretest	Pretest Lesson	80%
Phase 2	Teach multiplication with regrouping at the concrete level	Lessons 1-4	100% 2/2 independent practice items
Phase 3	Teach multiplication with regrouping at the representational level	Lessons 5-8	100% 2/2 independent practice items
Phase 4	Teach RENAME Strategy applied to multiplication with regrouping	Lesson 9	100% accuracy
Phase 5	Teach multiplication with regrouping at the abstract level	Lesson 10	80% 4/5 independent practice items
Phase 6	Teach FAST RENAME Strategy for solving word problems	Lesson 11	100% accuracy
	-	Lessons 12-15	80% 4/5
Phase 7	Teach computation and problem solving at the abstract level, differentiating between operations		independent practice items
	within word problems. Teacher guidance fades.	Lessons 16-18	88% 7/8
			independent practice items
Phase 8	Posttest	Posttest	90%
		Lesson	